Computer Graphics

- Light Transport -

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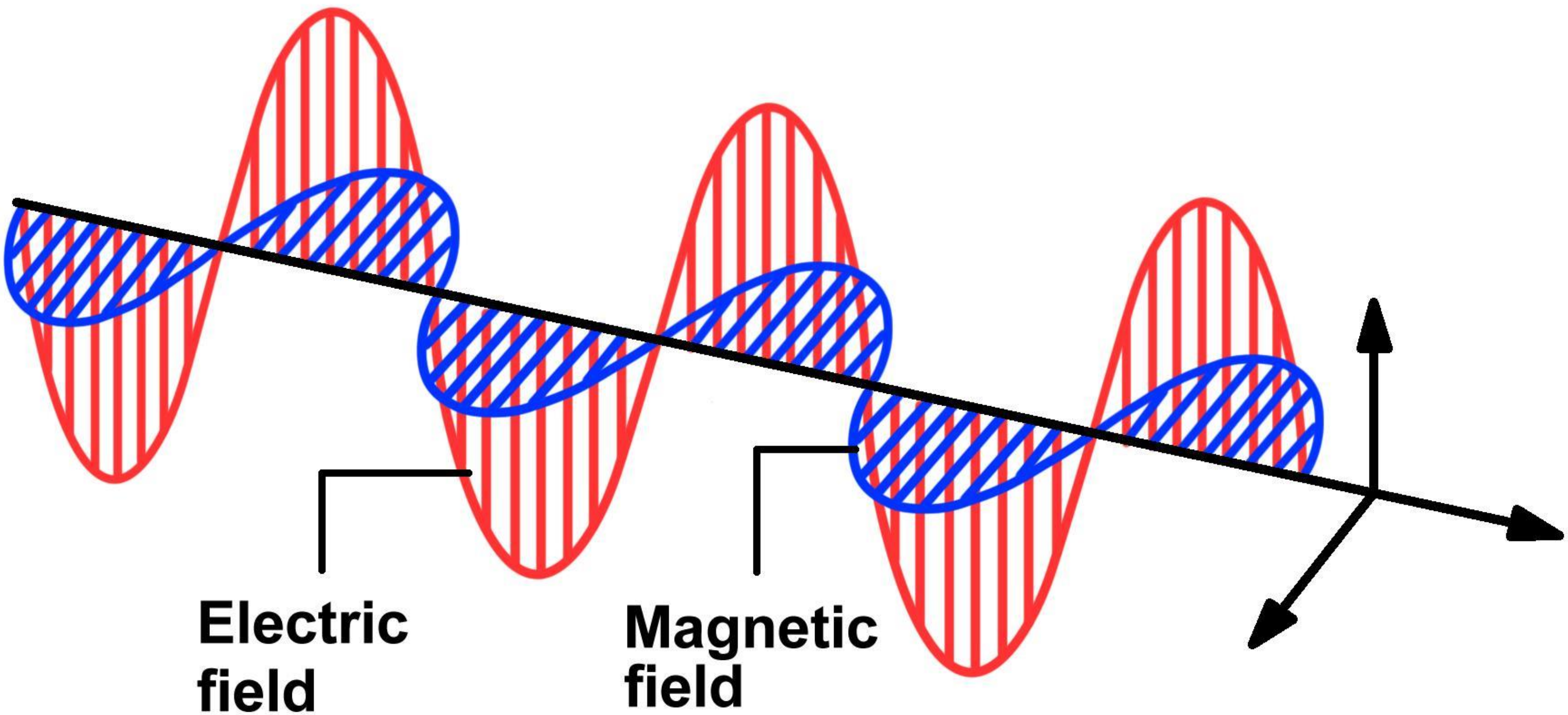
Overview

- **So far**
  - Nuts and bolts of ray tracing
- **Today**
  - Light
    - Physics behind ray tracing
    - Physical light quantities
    - Perception of light
    - Light sources
  - Light transport simulation
- **Next lecture**
  - Reflectance properties
  - Shading

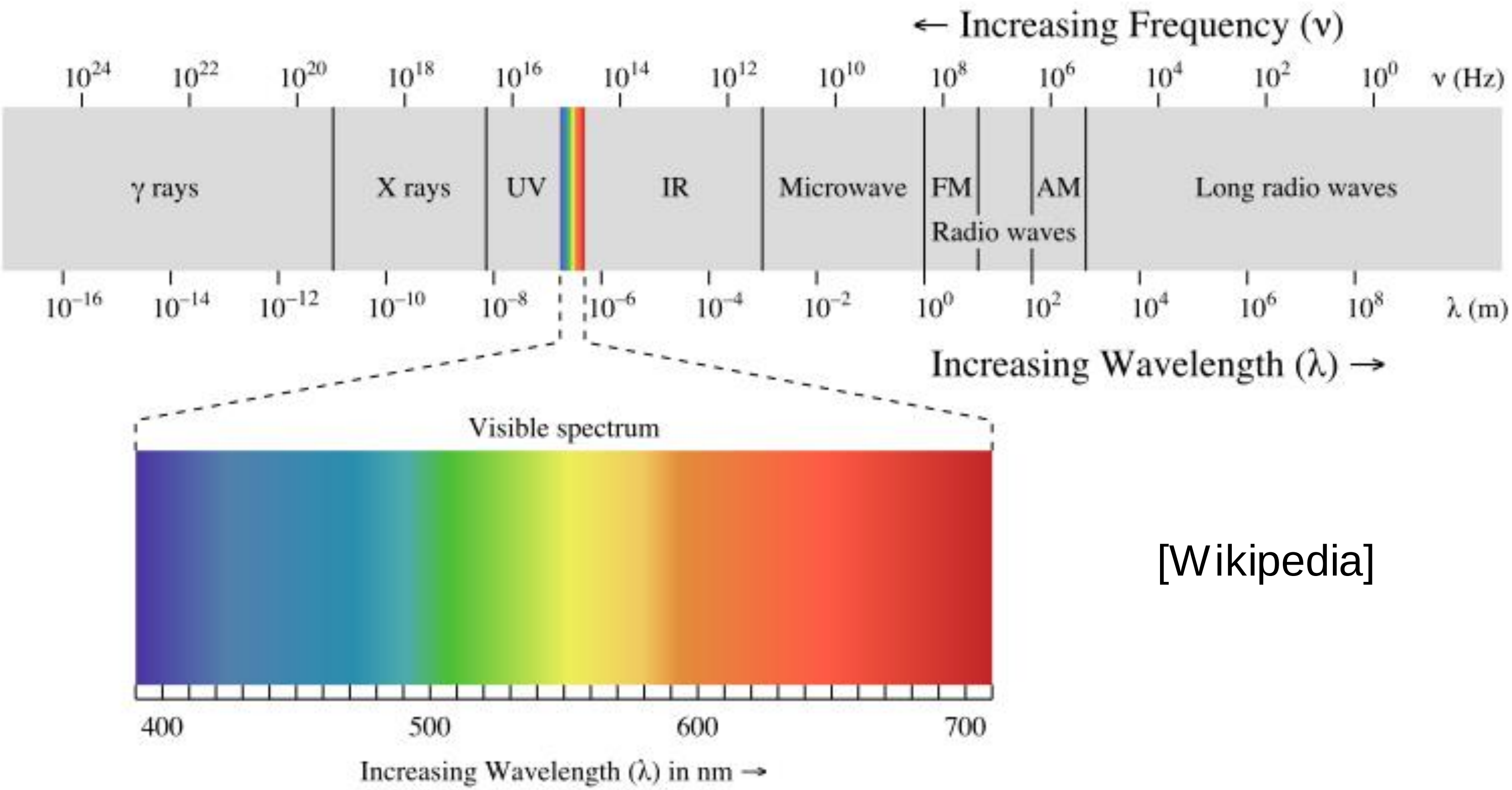
LIGHT

What is Light ?

- **Electro-magnetic wave propagating at speed of light**



What is Light ?



[Wikipedia]

What is Light ?

- **Ray**
  - Linear propagation
  - Geometrical optics
- **Vector**
  - Polarization
  - **Jones Calculus**: matrix representation
- **Wave**
  - Diffraction, interference
  - **Maxwell equations**: propagation of light
- **Particle**
  - Light comes in discrete energy quanta: photons
  - **Quantum theory**: interaction of light with matter
- **Field**
  - Electromagnetic force: exchange of virtual photons
  - **Quantum Electrodynamics (QED)**: interaction between particles

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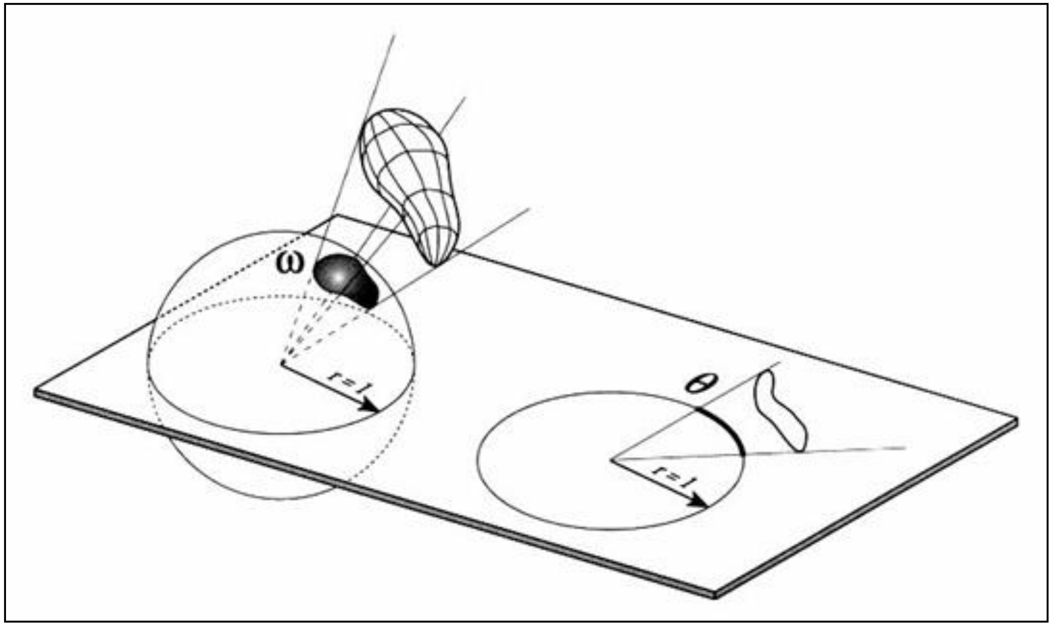
Light in Computer Graphics

- **Based on human visual perception**
  - Macroscopic geometry (→ Reflection Models)
  - Tristimulus color model (→ Human Visual System)
  - Psycho-physics: tone mapping, compression, … (→ RIS course)
- **Ray optic assumptions**
  - Macroscopic objects
  - Incoherent light
  - Light: scalar, real-valued quantity
  - Linear propagation
  - Superposition principle: light contributions add, do not interact
  - No attenuation in free space
- **Limitations**
  - No microscopic structures (≈ λ): diffraction, interference
  - No polarization
  - No dispersion, …

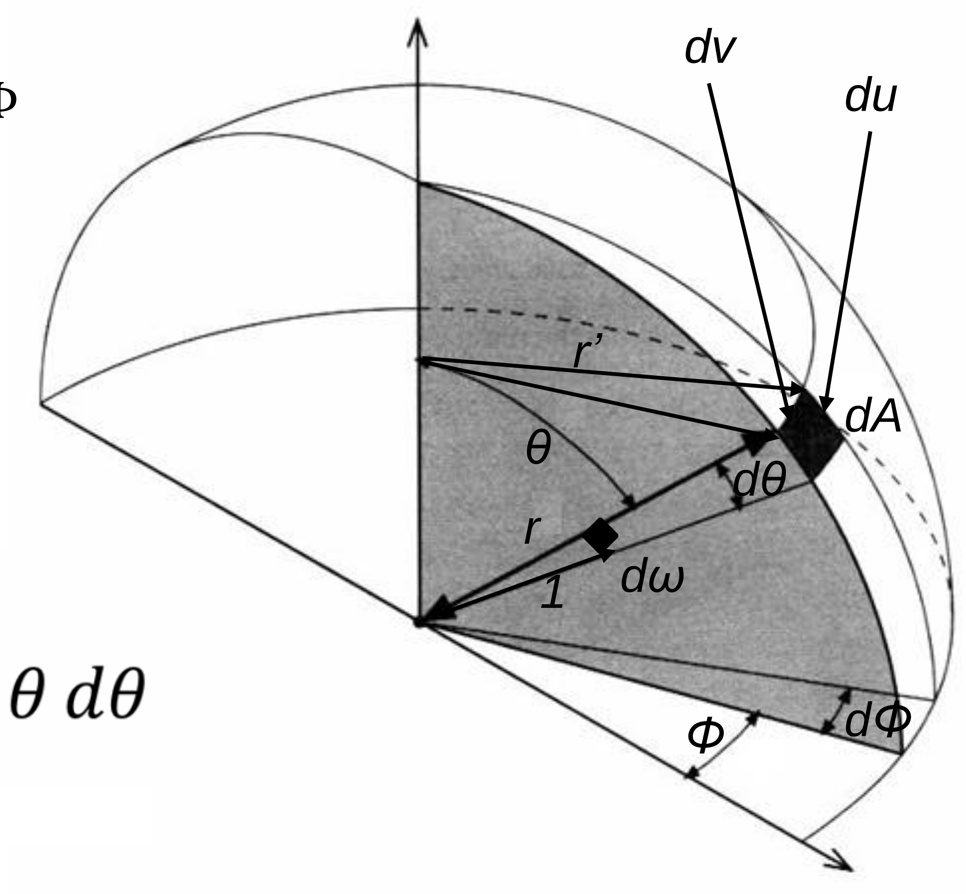
Angle and Solid Angle

- **The angle θ (in radians) subtended by a curve in the plane is the length of the corresponding arc on the unit circle: $l = \theta r = \theta$**
- **The solid angle Ω, $d\omega$ subtended by an object is the surface area of its projection onto the unit sphere**
  - Units for measuring solid angle: steradian [sr] (dimensionless)



Solid Angle in Spherical Coords

- **Infinitesimally small solid angle *dω***
  - $du = r\, d\theta$
  - $dv = r'\, d\Phi = r \sin\theta\, d\Phi$
  - $dA = du\, dv = r^2 \sin\theta\, d\theta d\Phi$
  - $d\omega = dA/r^2 = \sin\theta\, d\theta d\Phi$



- **Finite solid angle**

$$\Omega = \int_{\phi_0}^{\phi_1} d\phi \int_{\theta_0(\phi)}^{\theta_1(\phi)} \sin\theta\, d\theta$$

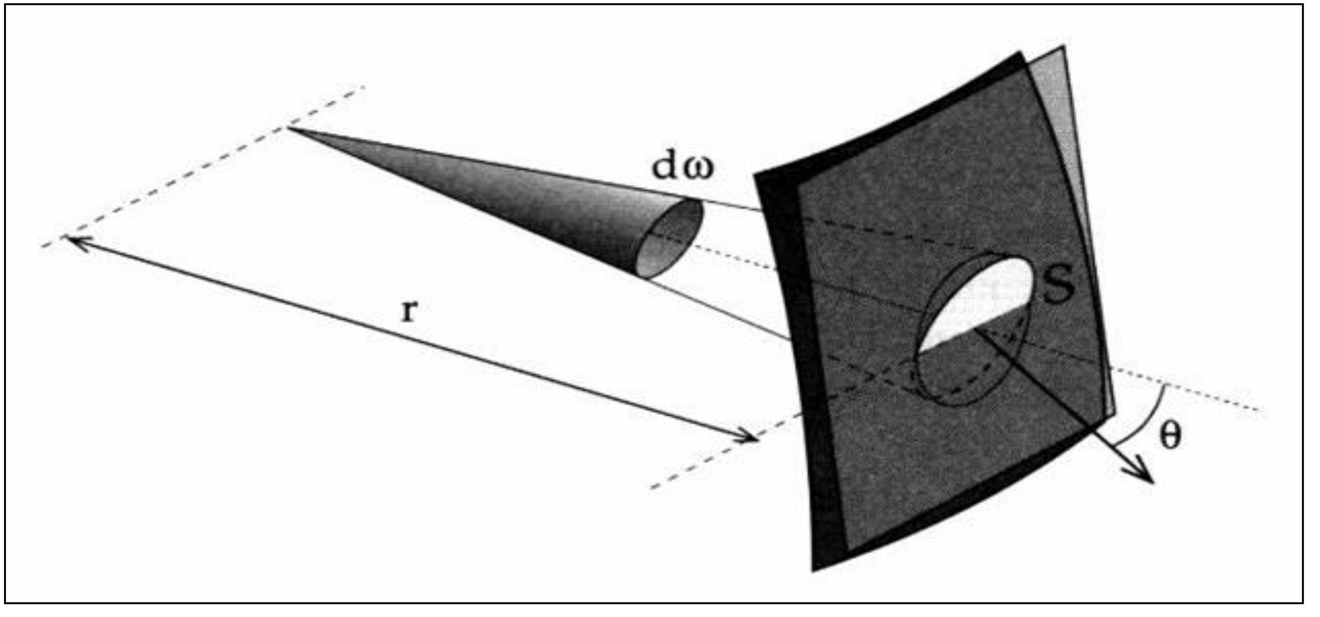
Solid Angle for a Surface

- The solid angle subtended by a small surface patch *S* with area *dA* is obtained (i) by projecting it orthogonal to the vector *r* from the origin:

$$dA \cos\theta$$

and (ii) dividing by the squared distance to the origin: $\mathrm{d}\omega = \frac{\mathrm{d}A\cos\theta}{r^2}$

$$\Omega = \iint_S \frac{\vec{r}\cdot\vec{n}}{r^3} dA$$



Radiometry

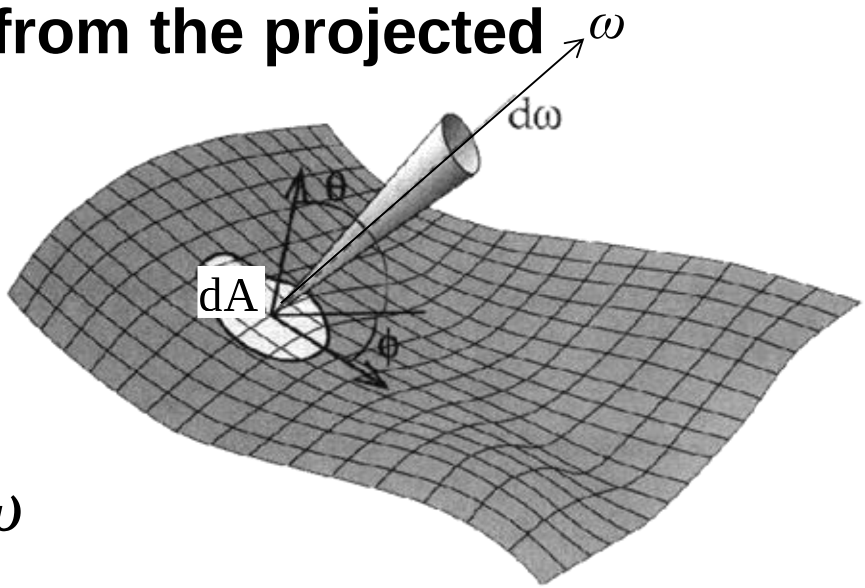
- **Definition:**
  - Radiometry is the science of measuring radiant energy transfers. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral photometers.

- **Radiometric Quantities**

– Energy	[J]	*Q*	(#Photons x Energy = $n \cdot h\nu$)
– Radiant power	[watt = J/s]	Φ	(Total Flux)
– Intensity	[watt/sr]	*I*	(Flux from a point per s.angle)
– Irradiance	[watt/m^2]	*E*	(Incoming flux per area)
– Radiosity	[watt/m^2]	*B*	(Outgoing flux per area)
– Radiance	[watt/(m^2 sr)]	*L*	(Flux per area & proj. s. angle)

Radiometric Quantities: Radiance

- **Radiance is used to describe radiant energy transfer**
- **Radiance *L* is defined as**
  - The power (flux) traveling through some point *x*
  - In a specified direction $\omega = (\theta, \varphi)$
  - Per unit area perpendicular to the direction of travel
  - Per unit solid angle
- **Thus, the differential power $d^2\Phi$ radiated through the differential solid angle $d\omega$, from the projected differential area $dA\cos\theta$ is:**



$$d^2\Phi = L(x,\omega)\, dA \cos\theta \, d\omega$$

Radiometric Quantities: Irradiance

- Irradiance E is defined as the **total power per unit area** (flux density) incident onto a surface. To obtain the total flux incident to *dA*, the incoming radiance L_i is integrated over the upper hemisphere Ω_+ above the surface:

$$E \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega_+} L_i(x,\omega)\cos\theta\, d\omega\right] dA$$

$$E(x) = \int_{\Omega_+} L_i(x,\omega)\cos\theta\, d\omega = \int_0^{\frac{\pi}{2}}\int_0^{2\pi} L_i(x,\omega)\cos\theta\sin\theta\, d\theta d\phi$$

Radiometric Quantities: Radiosity

- **Radiosity B** is defined as the **total power per unit area** (flux density) **exitant from** a surface. To obtain the total flux incident to dA, the outgoing radiance L_o is integrated over the upper hemisphere Ω_+ above the surface**:**

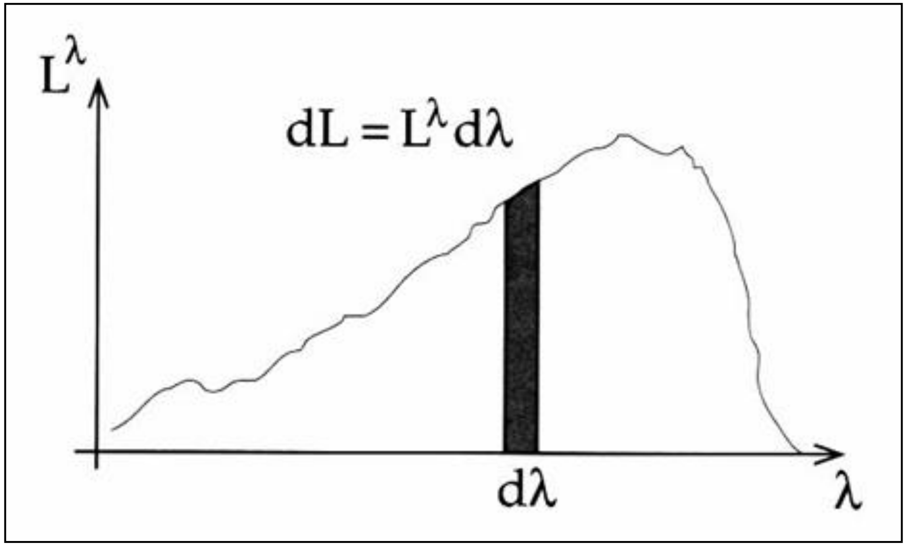
$$B \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega_+} L_o(x,\omega)\cos\theta\, d\omega\right] dA$$

$$B(x) = \int_{\Omega_+} L_o(x,\omega)\cos\theta\, d\omega = \int_0^{\frac{\pi}{2}}\int_0^{2\pi} L_o(x,\omega)\cos\theta\sin\theta\, d\theta d\phi$$

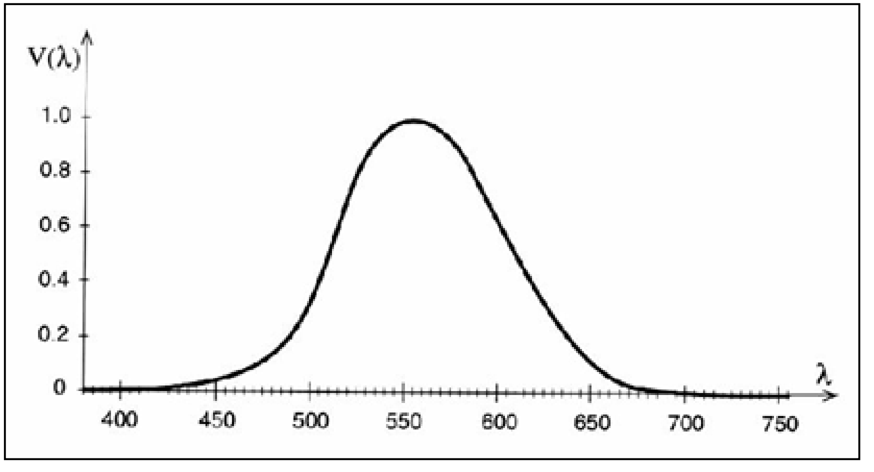
Spectral Properties

- **Wavelength**
  - Light is composed of electromagnetic waves
  - These waves have different frequencies and wavelengths
  - Most transfer quantities are continuous functions of wavelength
- **In graphics**
  - Each measurement *L(x,ω)* is for a discrete band of wavelength only
    - Often R(ed, long), G(reen, medium), B(lue, short) (but see later)



Photometry

- The human eye is sensitive to a limited range of wavelengths
  - Roughly from 380 nm to 780 nm
- Our visual system responds differently to different wavelengths
  - Can be characterized by the *Luminous Efficiency Function* V(λ)
  - Represents the average human spectral response
  - Separate curves exist for light and dark adaptation of the eye
- Photometric quantities are derived from radiometric quantities by *integrating* them against this function

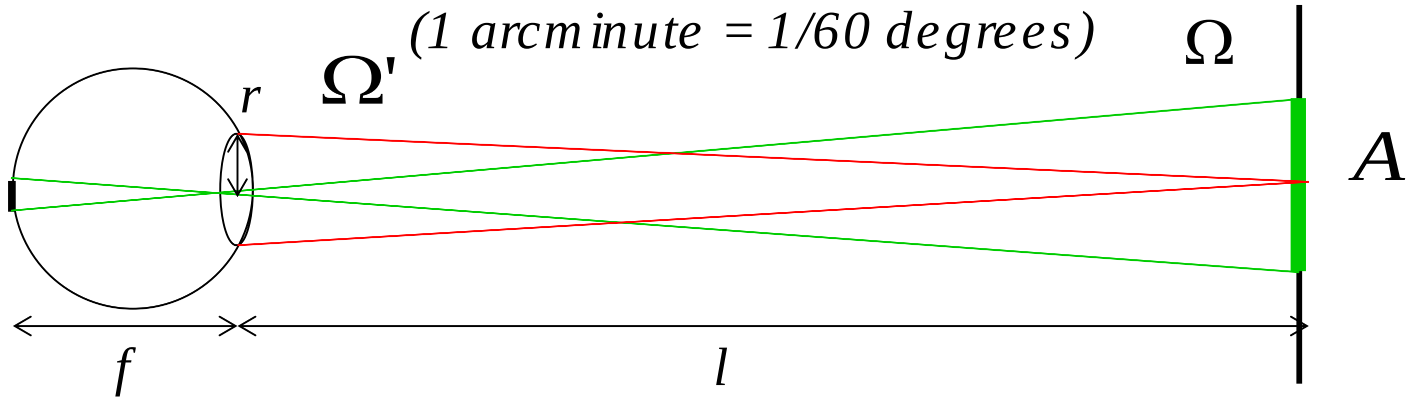


Radiometry vs. Photometry

Physics-based quantities **Perception-based quantities**

	Radiometry	→	Photometry	
W	Radiant power	→	Luminous power	Lumens (lm)
W/m^2	Radiosity Irradiance	→	Luminosity Illuminance	Lux (lm/m^2)
$W/m^2/sr$	Radiance	→	Luminance	cd/m^2 ($lm/m^2/sr$)

Perception of Light



photons / second = **flux = energy / time = power** (Φ)

rod sensitive to flux

angular extent of rod = resolution *(≈ 1 arcminute²)* Ω

projected rod size = area $A \approx l^2 \cdot \Omega$

angular extent of pupil aperture (r ≤ 4 mm) = solid angle $\Omega' \approx \pi \cdot r^2 / l^2$

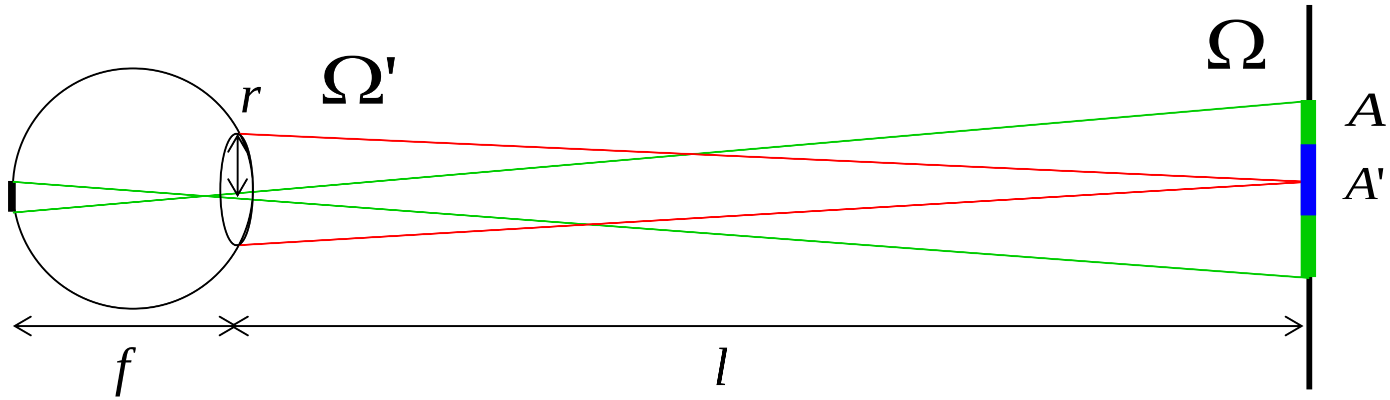
flux proportional to area and solid angle $\Phi = L\,A\,\Omega'$

radiance = *flux per unit area per unit solid angle* $L = \dfrac{\Phi}{\Omega' \cdot A}$

The eye detects radiance

As l increases: $\Phi_0 = L \cdot l^2 \cdot \Omega \cdot \pi \dfrac{r^2}{l^2} = L \cdot \text{const}$

Brightness Perception



- *A' > A : photon flux per rod stays constant*
- *A' < A : photon flux per rod decreases*

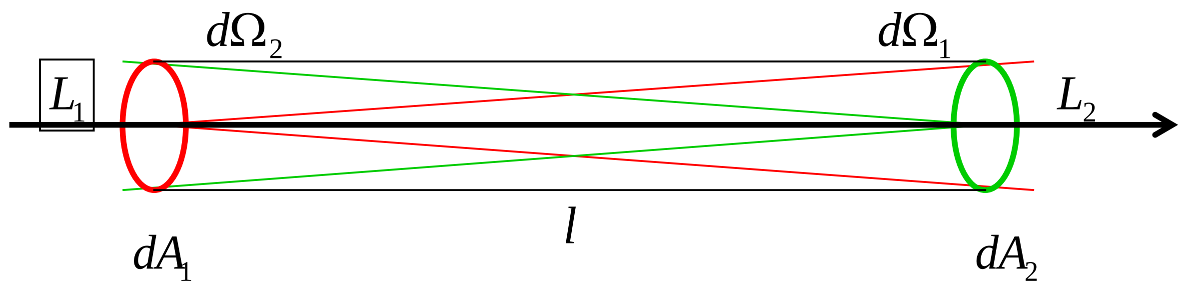
Where does the Sun turn into a star ?

– *Depends on apparent Sun disc size on retina*

– *Photon flux per rod stays the same on Mercury, Earth or Neptune*

– *Photon flux per rod decreases when* Ω' < 1 arcminute^2 *(beyond Neptune)*

Radiance in Space



Flux leaving surface 1 must be equal to flux arriving on surface 2

$$L_1 d\Omega_1 dA_1 = L_2 d\Omega_2 dA_2$$

From geometry follows $d\Omega_1 = \frac{dA_2}{l^2}$ $\quad d\Omega_2 = \frac{dA_1}{l^2}$

Ray throughput T: $T = d\Omega_1 \cdot dA_1 = d\Omega_2 \cdot dA_2 = \frac{dA_1 \cdot dA_2}{l^2}$

$$L_1 = L_2$$

The radiance *in the direction of a light ray* remains constant *as it propagates along the ray*

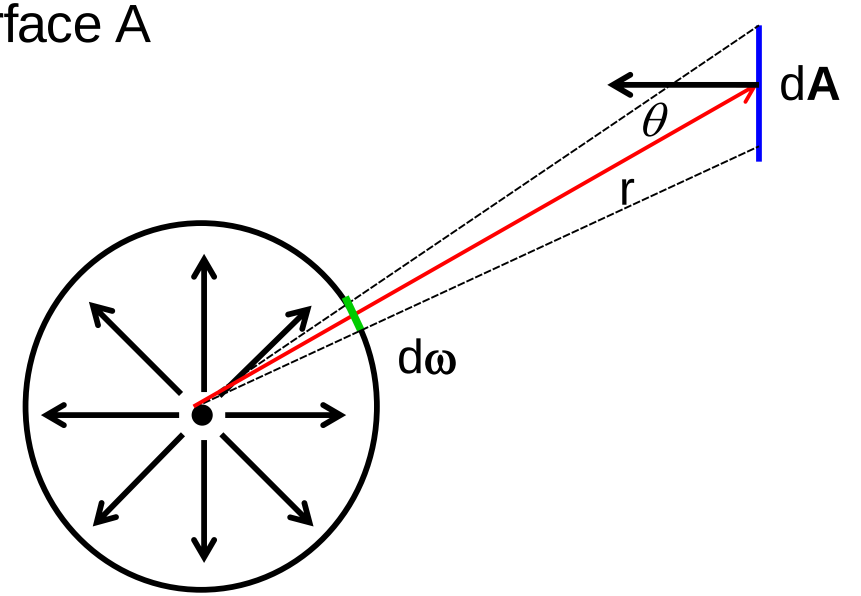
Point Light Source

- **Point light with *isotropic* radiance**
  - Power (total flux) of a point light source
    - Φ_g = Power of the light source [watt]
  - Intensity of a light source (radiance cannot be defined, no area)
    - $I = \Phi_g / 4\pi$ [watt/sr]
  - Irradiance on a sphere with radius r around light source:
    - $E_r = \Phi_g / (4 \pi r^2)$ [watt/m2]
  - Irradiance on some other surface A

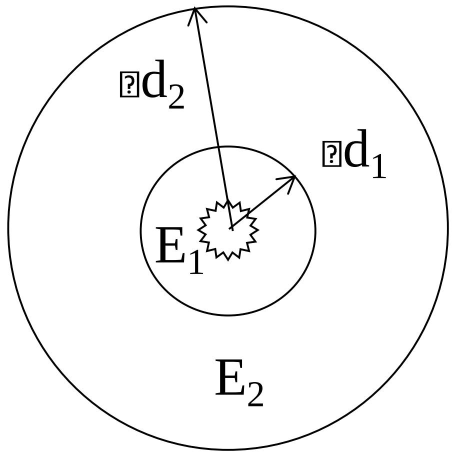
$$E(x) = \frac{d\Phi_g}{dA} = \frac{d\Phi_g}{d\omega}\frac{d\omega}{dA} = I\frac{d\omega}{dA}$$

$$= \frac{\Phi_g}{4\pi} \cdot \frac{dA\cos\theta}{r^2 dA}$$

$$= \frac{\Phi_g}{4\pi} \cdot \frac{\cos\theta}{r^2}$$



Inverse Square Law



Irradiance E:

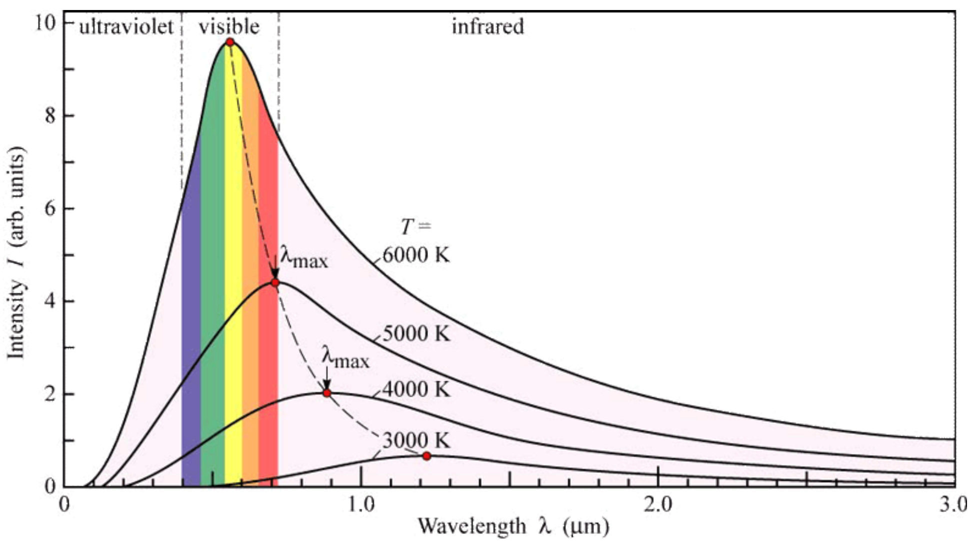
$$\frac{E_1}{E_2} = \frac{d_2^2}{d_1^2}$$

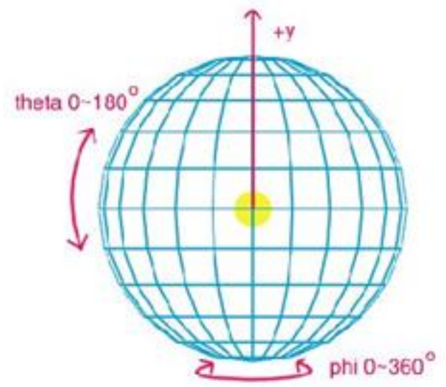
- **Irradiance *E*: power per m^2**
  - Illuminating quantity
- **Distance-dependent**
  - Double distance from emitter: area of sphere is four times bigger
- **Irradiance falls off with inverse of squared distance**
  - For point light sources (!)

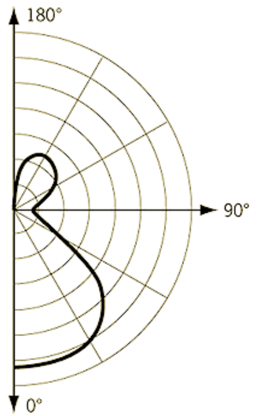
Light Source Specifications

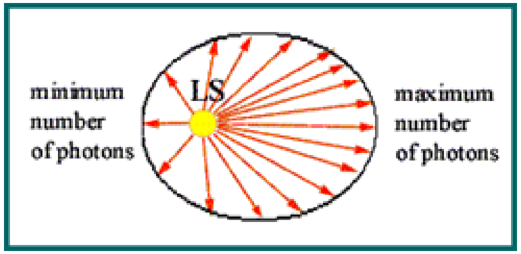
- **Power (total flux)**
  - Emitted energy / time
- **Active emission size**
  - Point, line, area, volume
- **Spectral distribution**
  - Thermal, line spectrum
- **Directional distribution**
  - Goniometric diagram

Black body radiation (see later)









Light Source Classification

Radiation characteristics

- **Directional light**
  - Spot-lights
  - Projectors
  - Distant sources
- **Diffuse emitters**
  - Torchieres
  - Frosted glass lamps
- **Ambient light**
  - "Photons everywhere"

Emitting area

- **Volume**
  - Neon advertisements
  - Sodium vapor lamps
- **Area**
  - CRT, LCD display
  - (Overcast) sky
- **Line**
  - Clear light bulb, filament
- **Point**
  - Xenon lamp
  - Arc lamp
  - Laser diode

Sky Light

- **Sun**
  - Point source (approx.)
  - White light (by def.)
- **Sky**
  - Area source
  - Scattering: blue
- **Horizon**
  - Brighter
  - Haze: whitish
- **Overcast sky**
  - Multiple scattering in clouds
  - Uniform grey
- **Several sky models are available**

Courtesy Lynch & Livingston

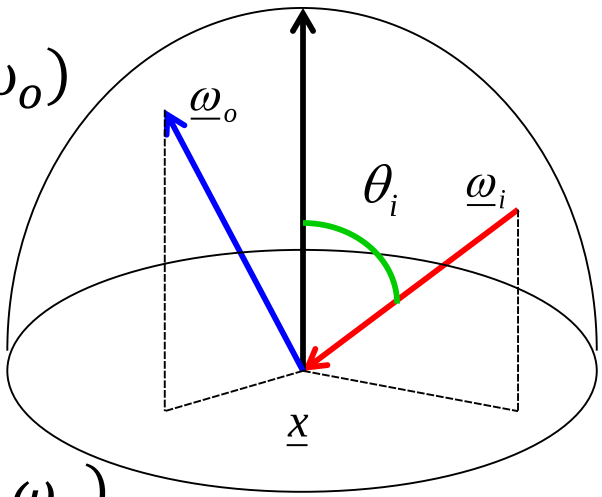
LIGHT TRANSPORT

Light Transport in a Scene

- **Scene**
  - Lights (emitters)
  - Object surfaces (partially absorbing)
- **Illuminated object surfaces become emitters, too!**
  - Radiosity = Irradiance minus absorbed photons flux density
    - Radiosity: photons per second per m^2 leaving surface
    - Irradiance: photons per second per m^2 incident on surface
- **Light bounces between all mutually visible surfaces**
- **Invariance of radiance in free space**
  - No absorption in-between objects
- **Dynamic energy equilibrium**
  - Emitted photons = absorbed photons (+ escaping photons)
    → Global Illumination, discussed in RIS lecture

Surface Radiance

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos\theta_i \, d\omega_i$$



- **Visible surface radiance** $L(x, \omega_o)$
  - Surface position x
  - Outgoing direction ω_o
- **Incoming illumination direction** ω_i

- **Self-emission** $L_e(x, \omega_o)$
- **Reflected light**
  - Incoming radiance from all directions $L_i(x, \omega_i)$
  - Direction-dependent reflectance (BRDF: bidirectional reflectance distribution function) $f_r(\omega_i, x, \omega_o)$

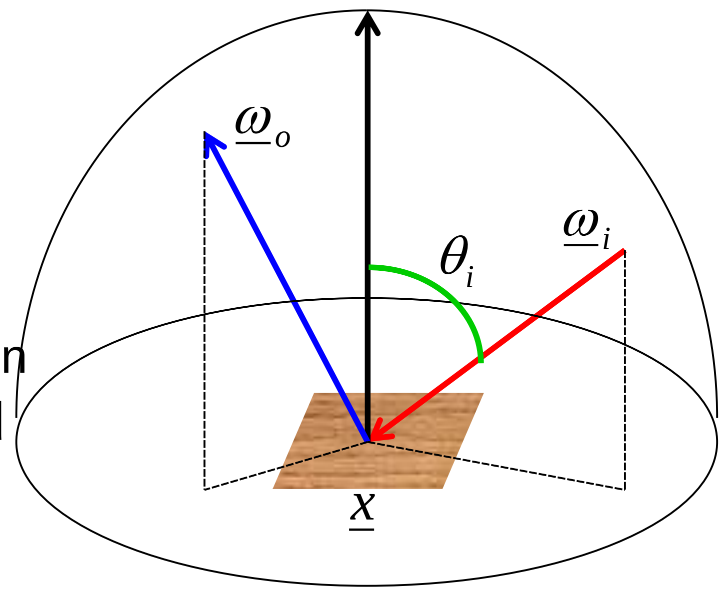
Rendering Equation

Typ. Exam Question!

- **Most important equation for graphics**
  - Expresses energy equilibrium in scene

$$L(x,\omega_o) = L_e(x,\omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

total radiance = emitted + reflected radiance



- **First term: emissivity of the surface**
  - Non-zero only for light sources
- **Second term: reflected radiance**
  - Integral over all possible incoming directions of radiance times angle-dependent surface reflection function
- **Fredholm integral equation of 2nd kind**
  - Unknown radiance appears both on the left-hand side and inside the integral
  - Numerical methods necessary to compute approximate solution

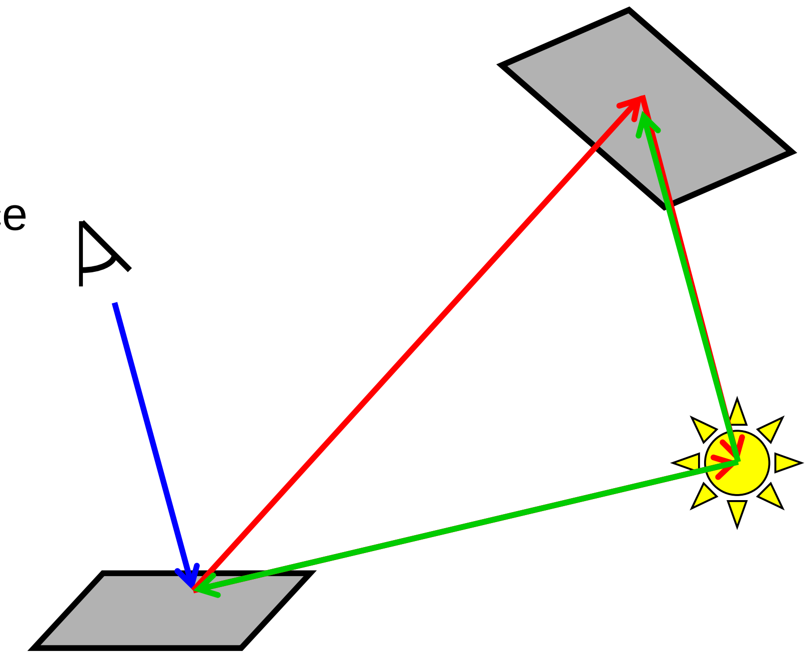
Rendering Equation: Approximations

- **Approximations based only on empirical foundations**
  - An example: polygon rendering in OpenGL
- **Using RGB instead of full spectrum**
  - Follows roughly the eye's sensitivity
- **Sampling hemisphere along finite, discrete directions**
  - Simplifies integration to summation
- **Reflection function model (BRDF)**
  - Parameterized function
    - Ambient: constant, non-directional, background light
    - Diffuse: light reflected uniformly in all directions
    - Specular: light from mirror-reflection direction

Ray Tracing

$$L(x,\omega_o) = L_e(x,\omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

- **Simple ray tracing**
  - Illumination from discrete point light sources only – direct illumination only
    - Integral → sum
    - No global illumination
  - Evaluates angle-dependent reflectance function (BRDF) – **shading process**
- **Advanced ray tracing techniques**
  - Recursive ray tracing
    - Multiple reflections/refractions (for specular surfaces)
  - Ray tracing for global illumination
    - Stochastic sampling (Monte Carlo methods)
    - Photon mapping



RE: Integrating over Surfaces

- **Outgoing illumination at a point**

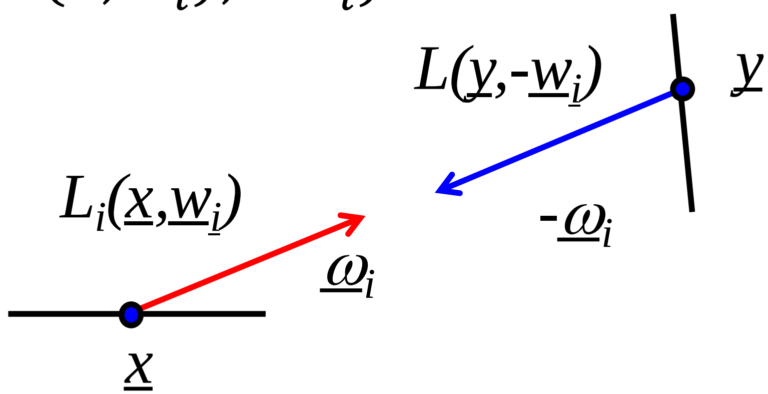
$$L(x, \omega_o) = L_e(x, \omega_o) + L_r(x, \omega_o)$$

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos\theta_i \, d\omega_i$$

- **Linking with other surface points**
  - Incoming radiance at x is outgoing radiance at y

$$L_i(x, \omega_i) = L(y, -\omega_i) = L(RT(x, \omega_i), -\omega_i)$$

  - Ray-Tracing operator: $y = RT(x, \omega_i)$



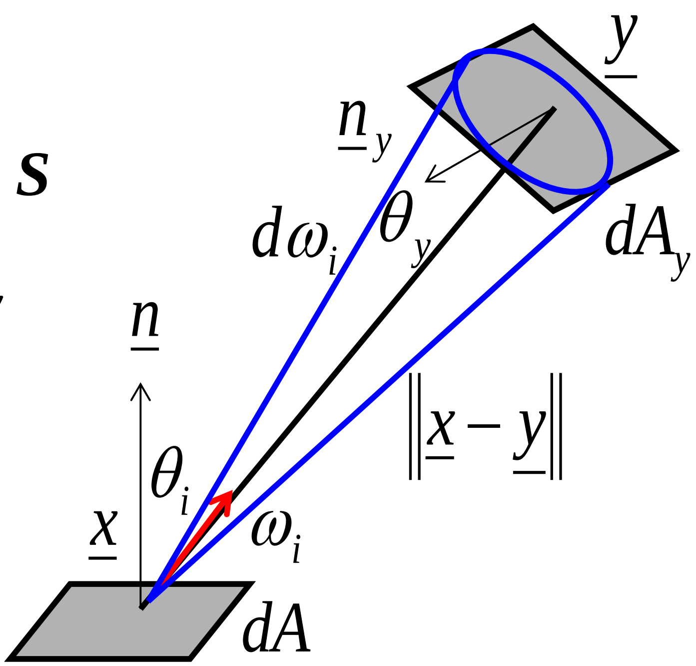
Integrating over Surfaces

- **Outgoing illumination at a point**

$$L(x,\omega_o) = L_e(x,\omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

- **Re-parameterization over surfaces *S***

$$d\omega_i = \frac{\cos\theta_y}{\|x-y\|^2} dA_y$$



$$L(x,\omega_o) = L_e(x,\omega_o) + \int_{y\in S} f_r(\omega(x,y), x, \omega_o) L_i(x, \omega(x,y)) V(x,y) \frac{\cos\theta_i \cos\theta_y}{\|x-y\|^2} dA_y$$

Integrating over Surfaces

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{y \in S} f_r(\omega(x,y), x, \omega_o) L_i(x, \omega(x,y)) V(x,y) \frac{\cos\theta_i \cos\theta_y}{\|x-y\|^2} dA_y$$

- **Geometry term:** $G(\mathrm{x},\mathrm{y}) = V(x,y) \frac{\cos\theta_i \cos\theta_y}{\|x-y\|^2}$

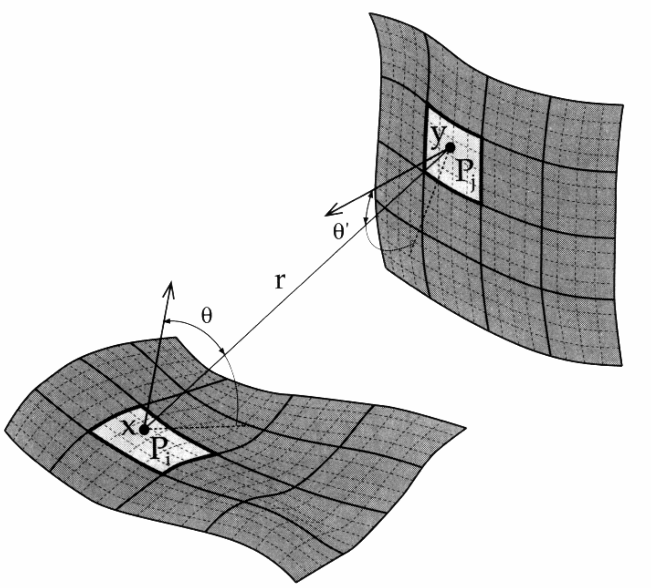
- **Visibility term:** $V(x,y) = \begin{cases} 1, & if\ visible \\ 0, & otherwise \end{cases}$

- **Integration over all surfaces:** $\int_{y \in S} \cdots dA_y$

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{y \in S} f_r(\omega(x,y), x, \omega_o) L_i\big(x, \omega(x,y)\big) G(x,y) dA_y$$

Radiosity Algorithm

- **Lambertian surface (only diffuse reflection)**
  - Radiosity equation: simplified form of the rendering equation
- **Dividing scene surfaces into small planar patches**
  - Assumes local constancy: diffuse reflection, radiosity, visibility
    - "Radiosity" algorithms: Discretizes into linear equation
- **Algorithm**
  - Form factor: percentage of light flowing between 2 patches
  - Form system of linear equations
  - Iterative solution
  - Discussed in details in RIS course



Radiosity Equation

- **Diffuse reflection ⇒ constant BRDF & emission**

$$f_r(\omega(x,y), x, \omega_o) = f_r(x) \Rightarrow$$

$$\rho(x) = \int_{\Omega_+} f_r(x) \cos\theta \, d\omega = f_r(x) \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \cos\theta \sin\theta \, d\theta d\phi = \pi f_r(x)$$

  - Reflectance factor or albedo: between [0,1]

- **Direction-independent out-going radiance**

$$L(x,\omega_o) = L_e(x,\omega_o) + f_r(x) \int_{y \in S} L_i\big(x, \omega(x,y)\big) G(x,y) dA_y$$

$$= L_e(x) + f_r(x)\, E(x) = L_o(x)$$

- **Form factor**
  - Defines percentage of light leaving *dAy* arriving at *dA*

$$F(x,y) = \frac{G(x,y)}{\pi}$$

Radiosity Equation

- **Radiosity**

$$B = \int_{\Omega_+} L_o(x, \omega_o) \cos\theta_o \, d\omega_o = L_o \int_{\Omega_+} \cos\theta_o \, d\omega_o = \pi L_o$$

$$B(x) = \pi L_e(x) + \pi f_r(x) E(x) = B_e(x) + \rho(x)\, E(x)$$

- **Irradiance**

$$E(x) = \int_{y \in S} L_i\big(x, \omega(x,y)\big) G(x,y) dA_y$$

$$= \int_{y \in S} L_o\big(y, -\omega(x,y)\big) G(x,y) dA_y = \int_{y \in S} \frac{B(y)}{\pi} G(x,y) dA_y$$

$$= \int_{y \in S} B(y) F(x,y) dA_y$$

Linear Operators

- **Properties**
  - Fredholm equation of 2nd kind
  - Global linking
    - Potentially each point with each other
    - Often sparse system (occlusions)
  - No consideration of volume effects!!

$$B(x) = B_e(x) + \rho(x) \int_{y \in S} F(x, y) B(y) \, dA_y$$

$$f(x) = g(x) + K[f(x)]$$

- **Linear operator**
  - Acts on functions like matrices act on vectors
  - Superposition principle
  - Scaling and addition

$$K[f(x)] = \int k(x, y) f(y) \, dy$$

$$K[af + bg] = aK[f] + bK[g]$$

Formal Solution of Integral Equations

$$B(x) = B_e(x) + \rho(x) \int_{y \in S} F(x,y) B(y)\, dA_y$$

- **Integral equation** $B(\cdot) = B_e(\cdot) + K[B(\cdot)] \Rightarrow (I - K)[B(\cdot)] = B_e(\cdot)$
- **Formal solution** $B(\cdot) = (I - K)^{-1}[B_e(\cdot)]$
- **Neumann series**
  – Converges only if $|K| < 1$ which is true in all physical settings

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots \qquad \frac{1}{I-K} = I + K + K^2 + \cdots$$

$$(I-K)\frac{1}{I-K} = (I-K)(I + K + K^2 + \cdots) = I + K + K^2 + \cdots - (K + K^2 + \cdots) = I$$

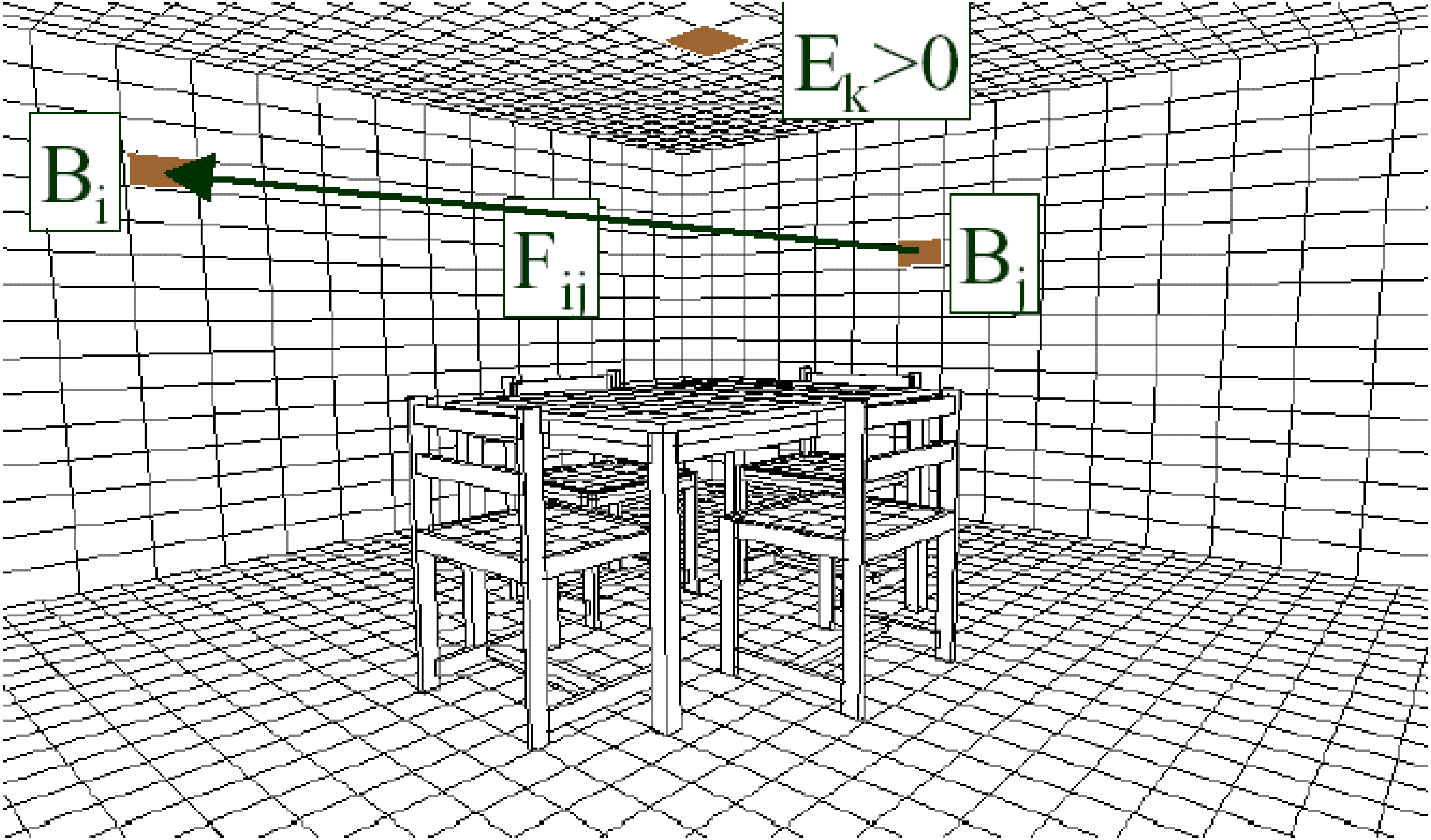
Formal Solutions (2)

- **Successive approximation**

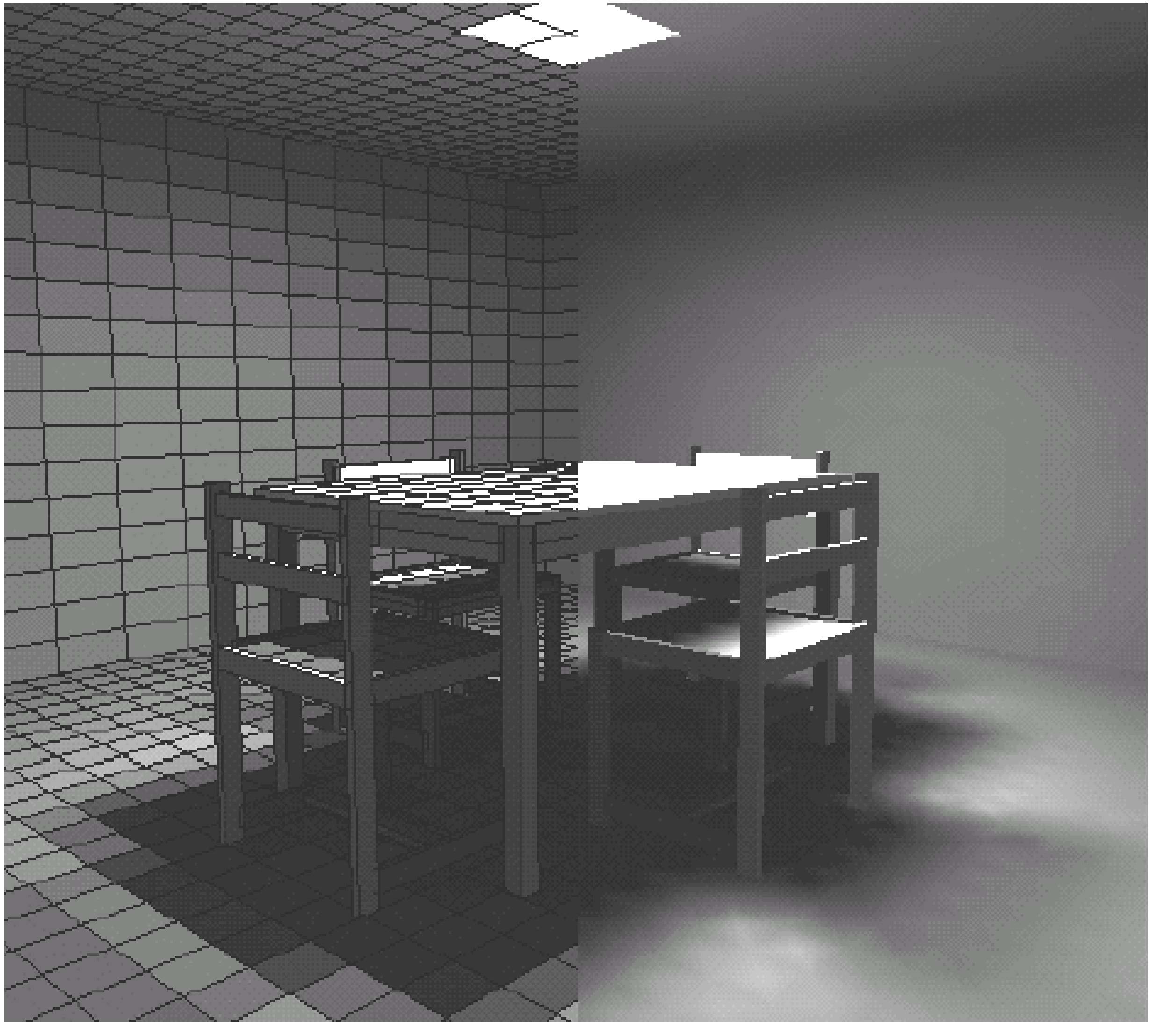
$$\frac{1}{I-K}B_e(\cdot) = B_e(\cdot) + K[B_e(\cdot)] + K^2[B_e(\cdot)] + \cdots$$
$$= B_e(\cdot) + K[B_e(\cdot) + K[B_e(\cdot) + \cdots]]$$

  - Direct light from the light source $B_1(\cdot) = B_e(\cdot)$
  - Light which is reflected and transported at most once $B_2(\cdot) = B_e(\cdot) + K[B_e(\cdot)]$
  - Light which is reflected and transported up to *n* times $B_n(\cdot) = B_e(\cdot) + K[B_{n-1}(\cdot)]$

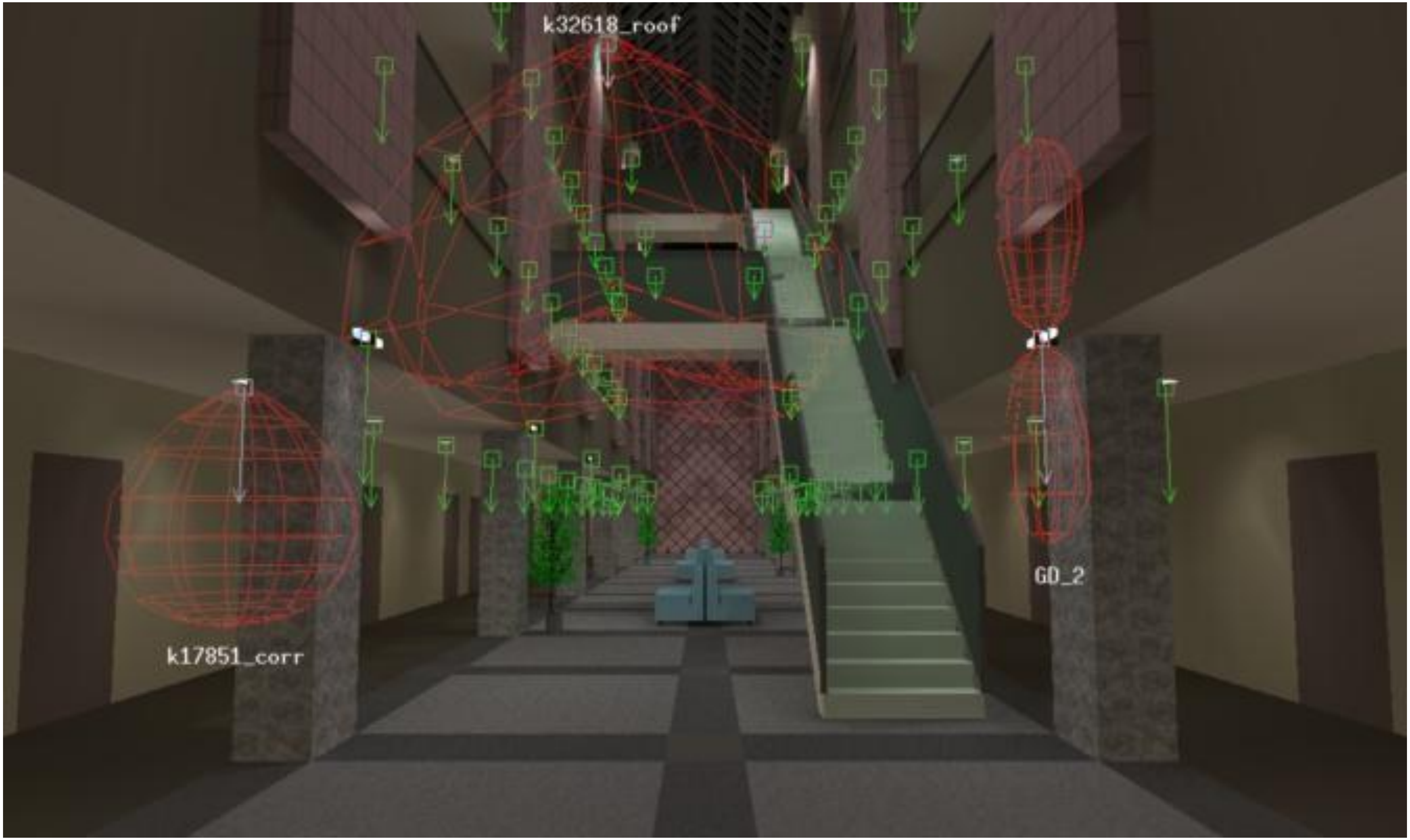
Radiosity Algorithm



Radiosity Algorithm



Lighting Simulation



Lighting Simulation

Lighting Simulation

Wrap Up

- **Physical Quantities in Rendering**
  - Radiance
  - Radiosity
  - Irradiance
  - Intensity
- **Light Perception**
- **Light Source Definition**
- **Rendering Equation**
  - Key equation in graphics (!)
  - Integral equation
  - Describes global balance of radiance