Computer Graphics

- Introduction to Ray Tracing -

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Rendering Algorithms

- **Rendering**
  - Definition: Given a 3D scene as input and a camera, generate a 2D image as a view from the camera of the 3D scene
- **Algorithms**
  - Ray Tracing
    - Declarative scene description
    - Physically-based simulation of light transport
  - Rasterization
    - Traditional procedural/imperative drawing of a scene content

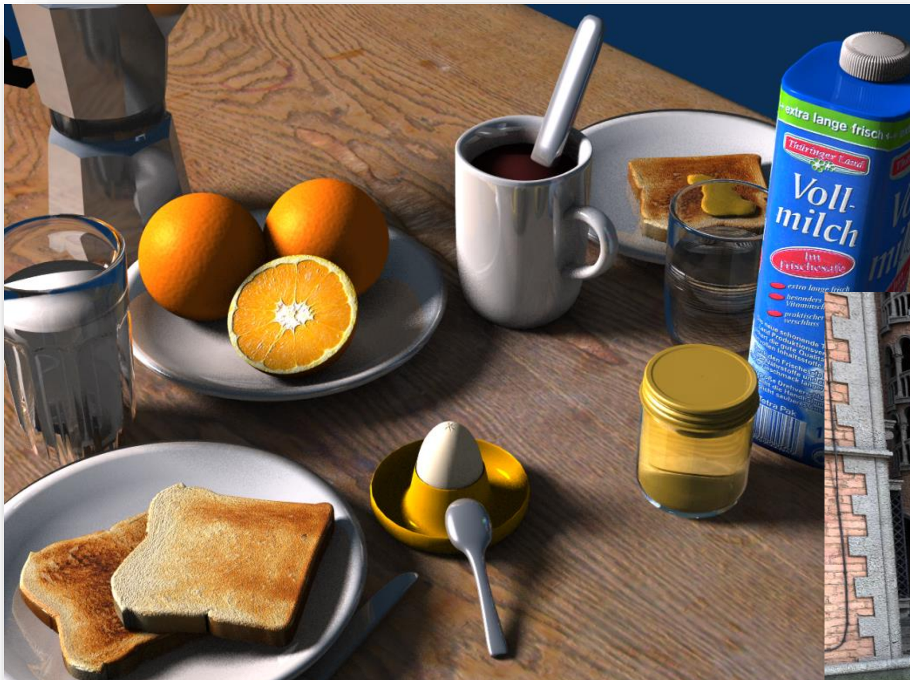
Scene

- **Surface Geometry**
  - 3D geometry of objects in a scene
  - Geometric primitives – triangles, polygons, spheres, …
- **Surface Appearance**
  - Color, texture, absorption, reflection, refraction, subsurface scattering
  - Mirror, glass, glossy, diffuse, …
- **Illumination**
  - Position and emission characteristics of light sources
  - Note: Light is reflected off of surfaces!
    - Secondary/indirect/global illumination
  - Assumption: air/empty space is totally transparent
    - Simplification that excludes scattering effects in participating media volumes
    - Later also volume objects, e.g. smoke, solid object (CT scan), …
- **Camera**
  - View point, viewing direction, field of view, resolution, …

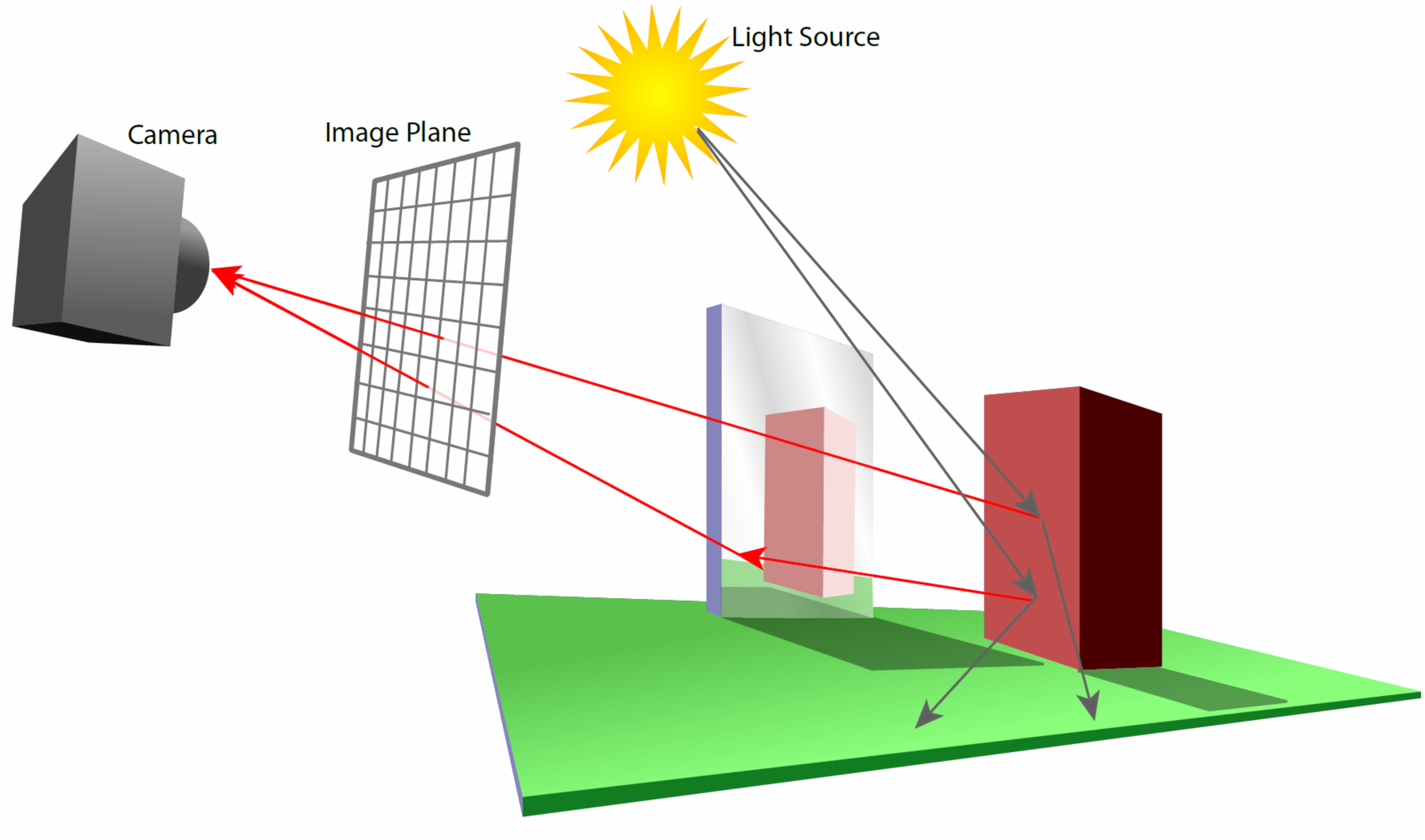
OVERVIEW OF RAY-TRACING

Ray Tracing Can…

- **Produce Realistic Images**
  - By simulating light transport



Light Transport (1)



Light Transport (2)

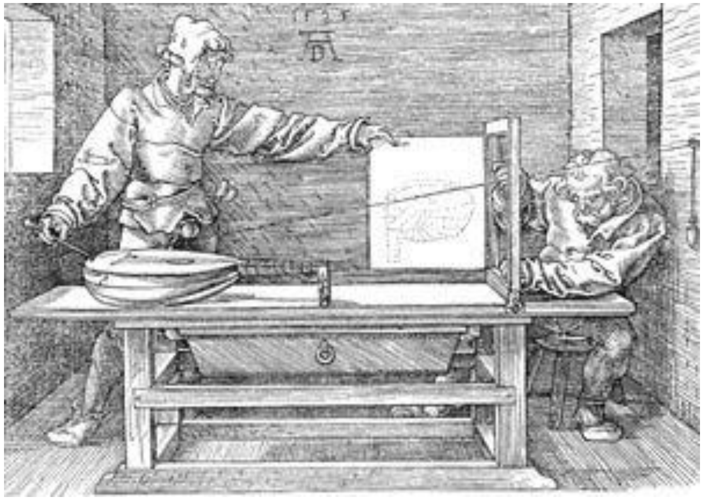
- **Light Distribution in a Scene**
  - Dynamic equilibrium

- **Forward Light Transport**
  - Shoot photons from the light sources into scene
  - Reflect at surfaces and record when a detector is hit
    - Photons that hit the camera produce the final image
    - Most photons will not reach the camera
  - Particle Tracing

- **Backward Light Transport**
  - Start at the detector (camera)
  - Trace only paths that might transport light towards it
    - May try to connect to occluded light sources
  - Ray Tracing

Ray Tracing Is…

- **Fundamental rendering algorithm**



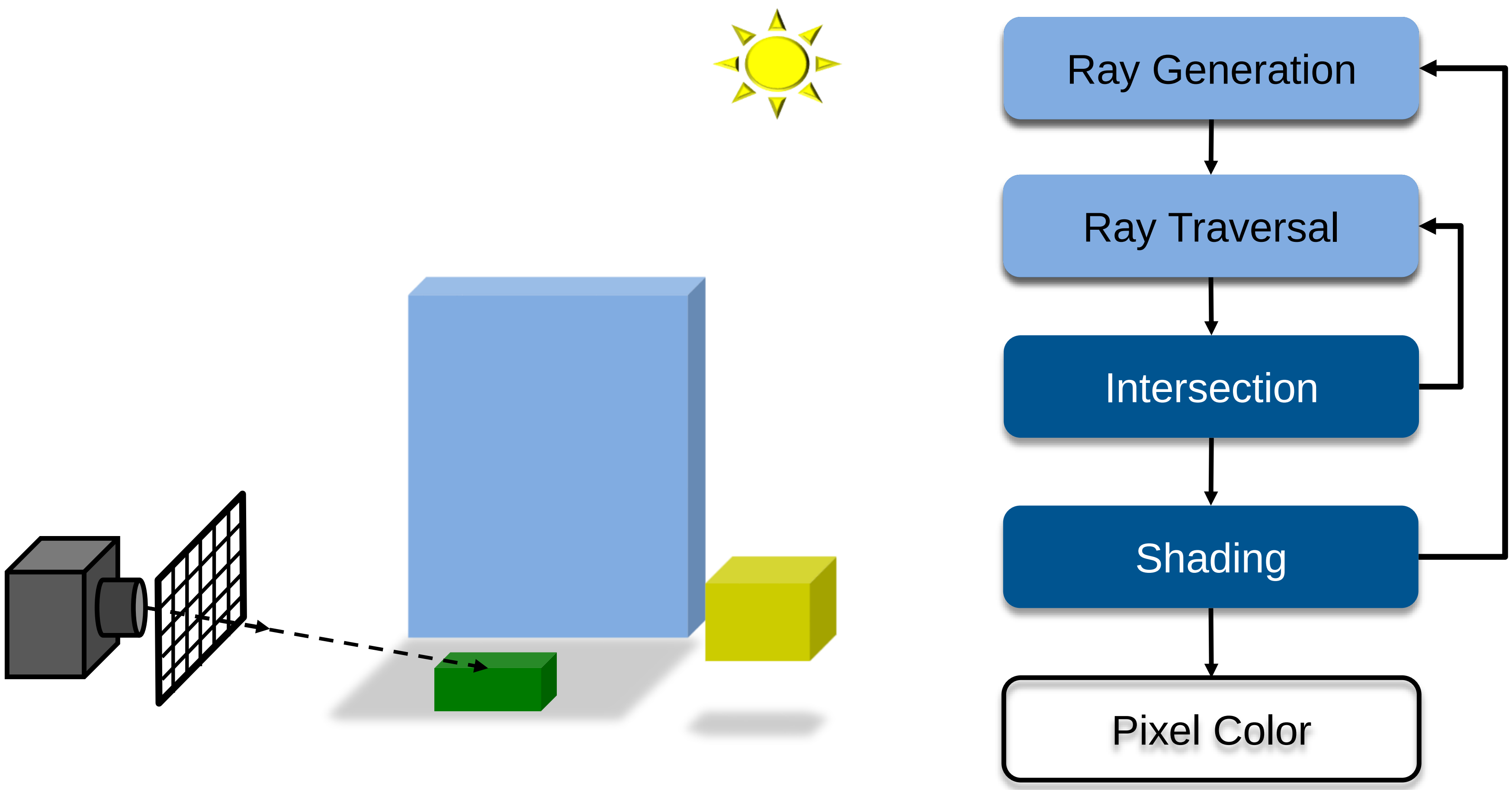
Perspective Machine, Albrecht Dürer

- **Automatic, simple and intuitive**
  - Easy to understand and implement
  - Delivers “correct“ images by default
- **Powerful and efficient**
  - Many optical global effects
  - Shadows, reflections, refractions, …
  - Efficient real-time implementation in SW and HW
  - Can work in parallel and distributed environments
  - Logarithmic scalability with scene size: O(log n) vs. O(n)
  - Output sensitive and demand driven
- **Concept of light rays is not new**
  - Empedocles (492-432 BC), Renaissance (Dürer, 1525), …
  - Uses in lens design, geometric optics, …

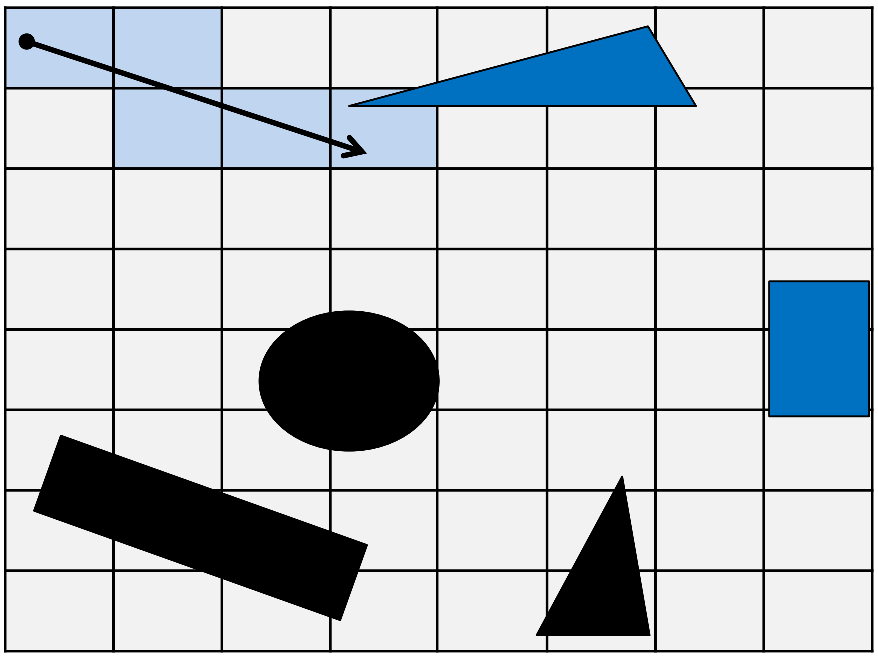
Fundamental Ray Tracing Steps

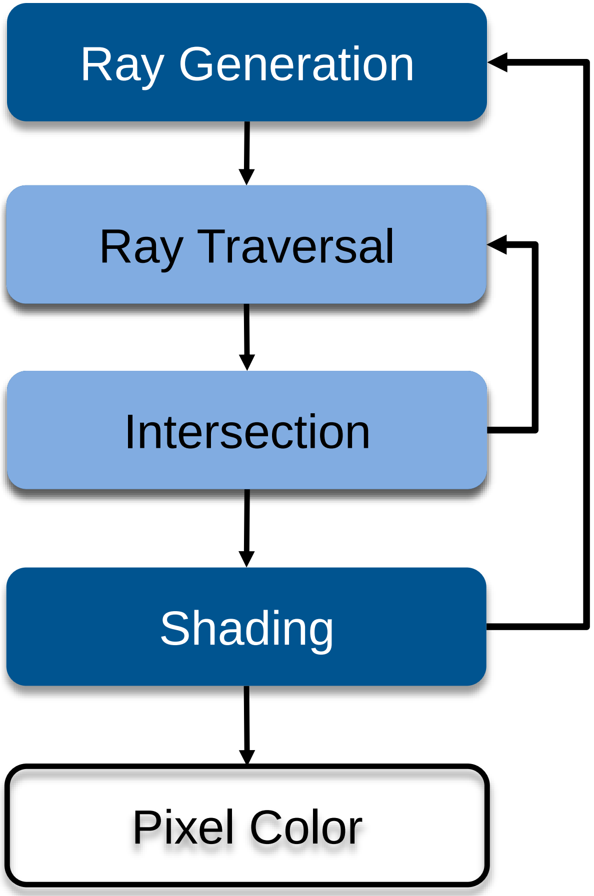
- **Generation of primary rays**
  - Rays from viewpoint along viewing directions into 3D scene
  - (At least) one ray per picture element (pixel)
- **Ray casting**
  - Traversal of spatial index structures
  - Ray-primitive intersection
- **Shading the hit point**
  - Determine pixel color
    - Energy (color) travelling along primary ray
  - Needed
    - Local material color, object texture and reflection properties
    - Local illumination at intersection point
      - Compute through recursive tracing of rays
      - Can be hard to determine correctly

Ray Tracing Pipeline (1)

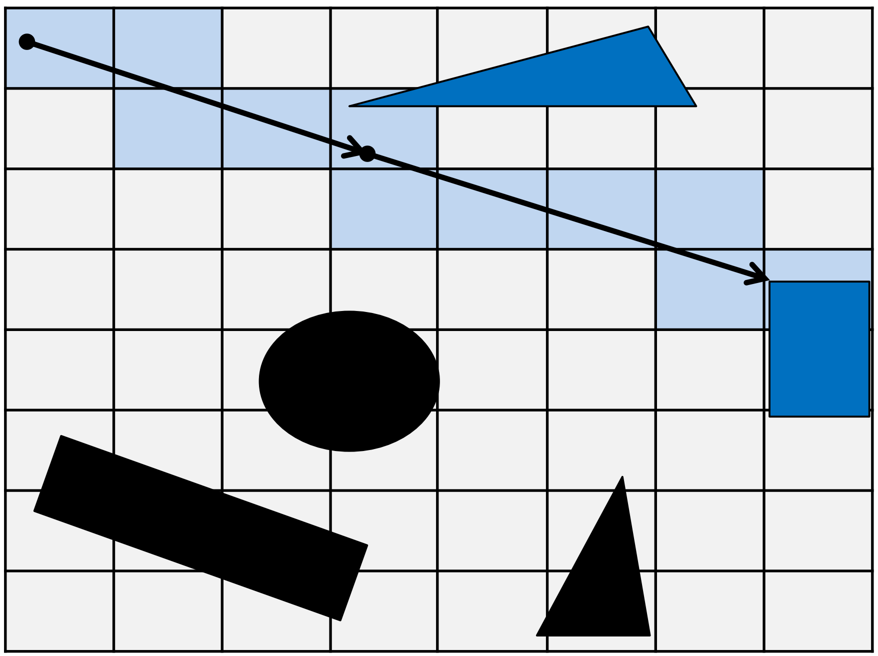


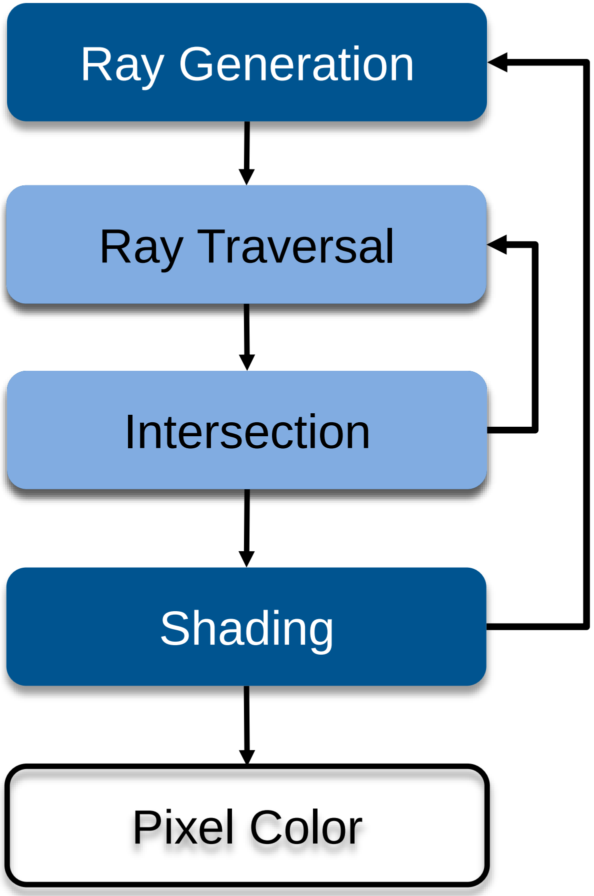
Ray Tracing Pipeline (2)



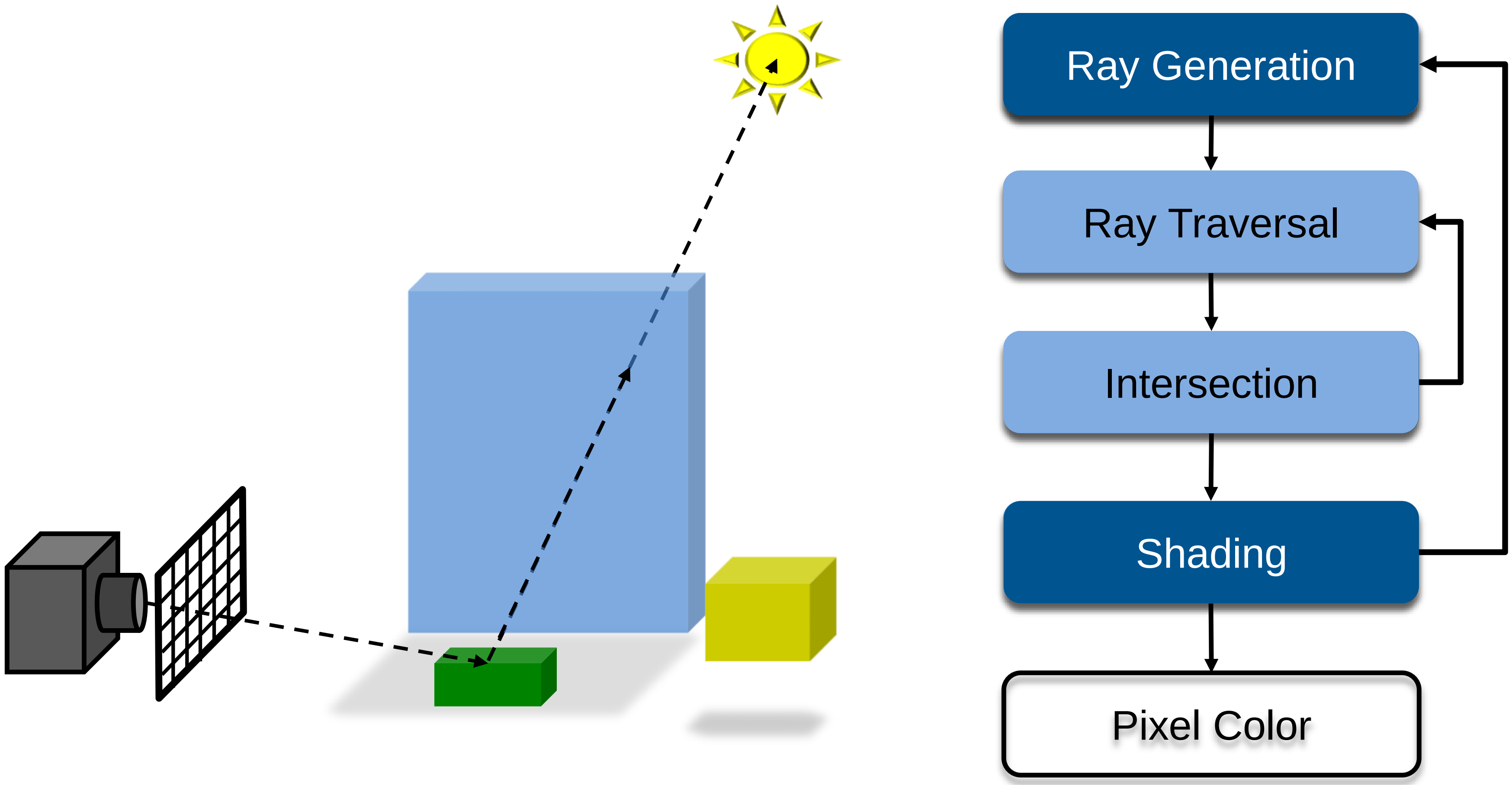


Ray Tracing Pipeline (3)

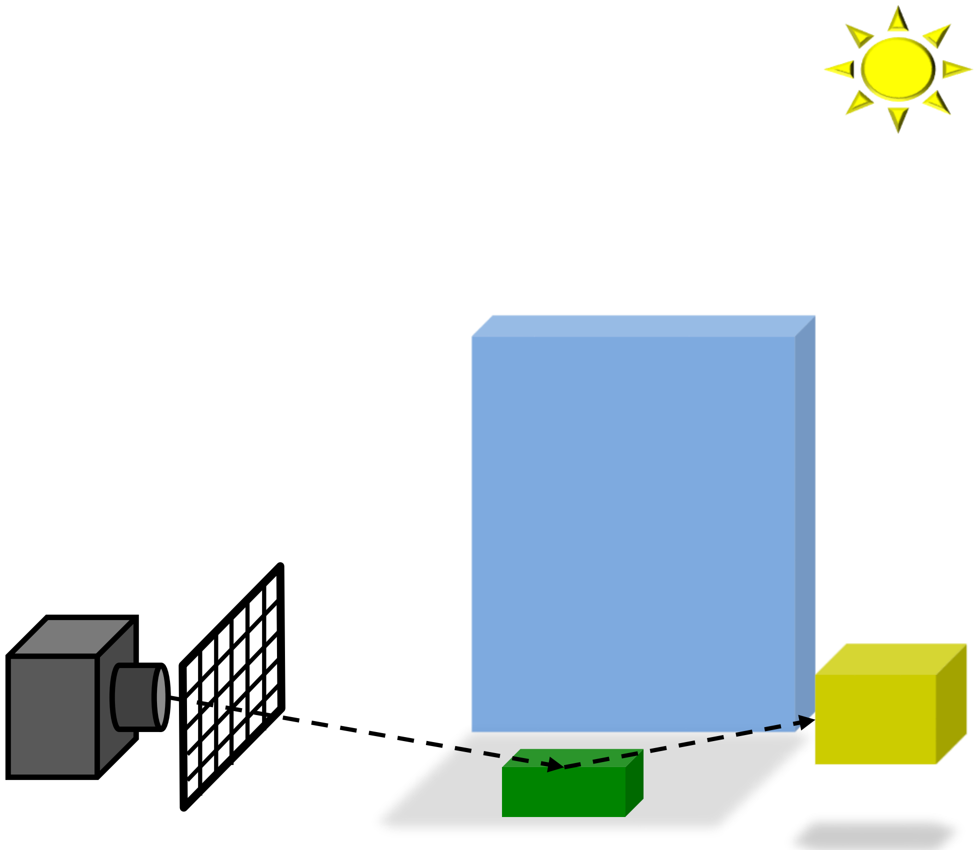


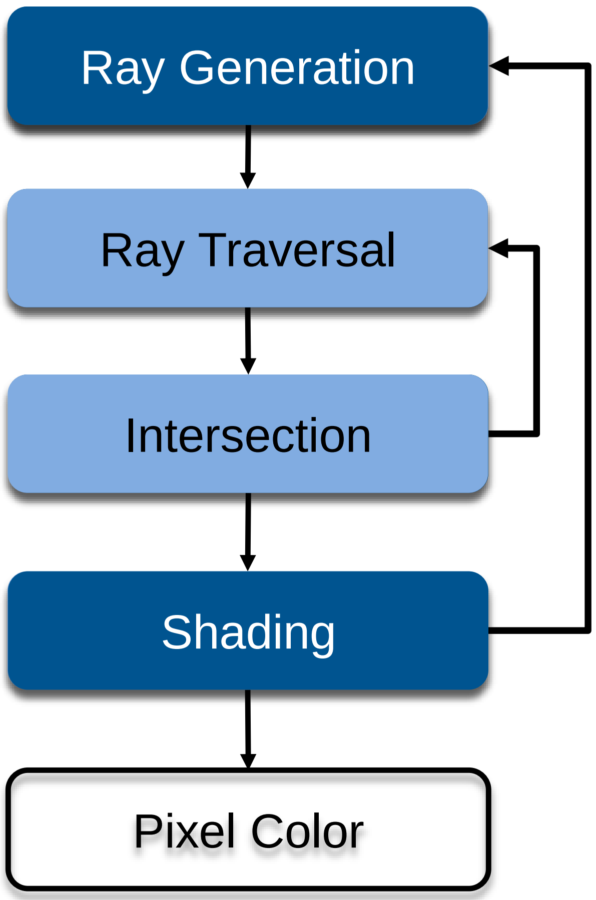


Ray Tracing Pipeline (4)

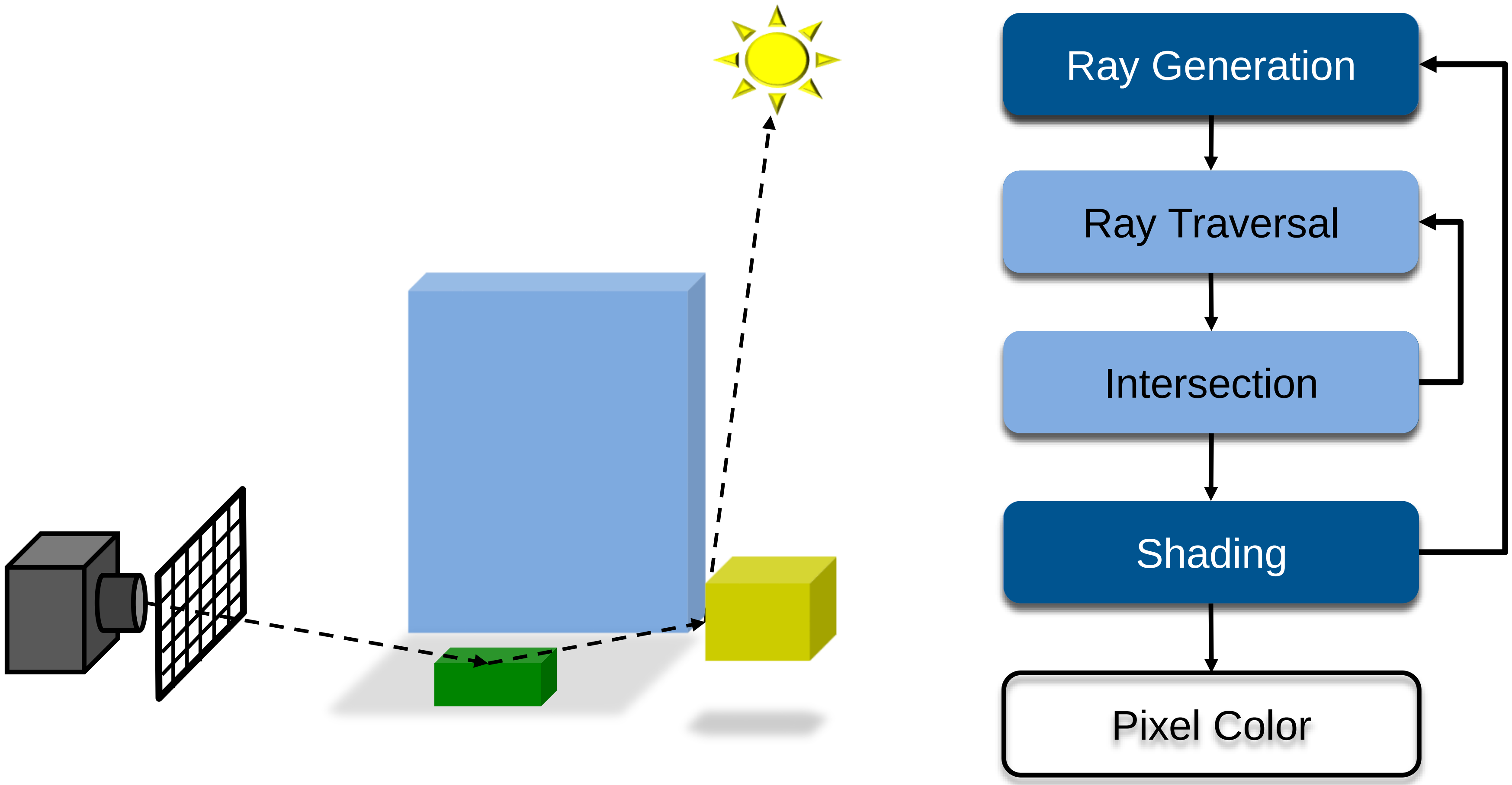


Ray Tracing Pipeline (5)

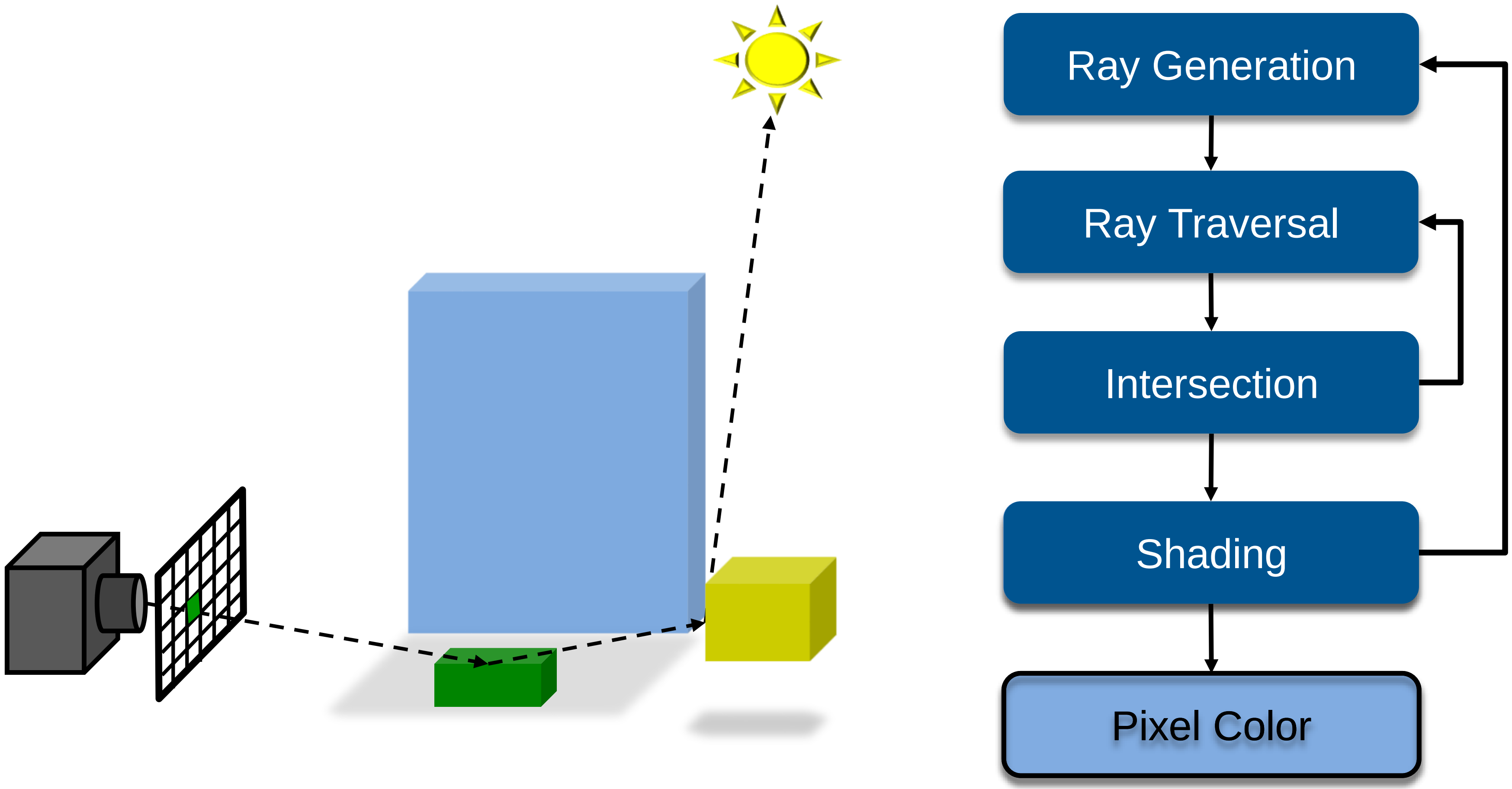




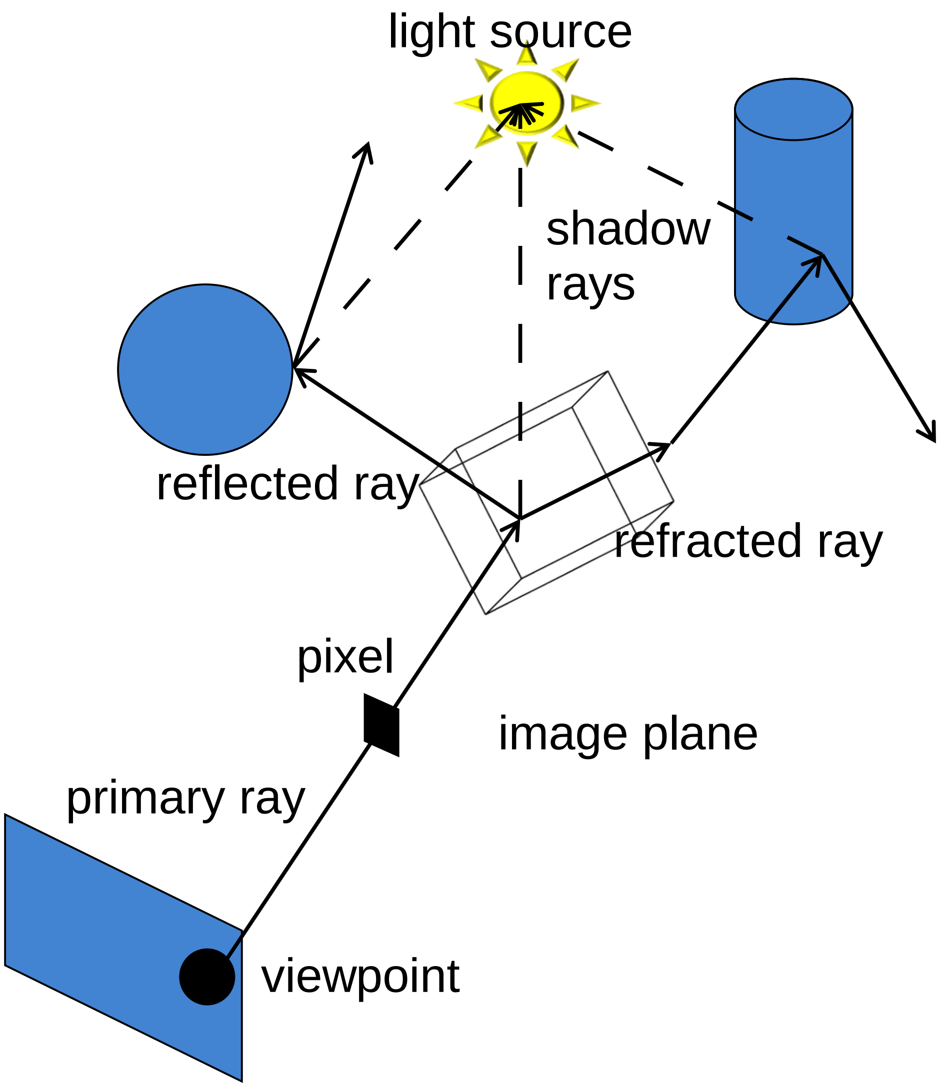
Ray Tracing Pipeline (6)



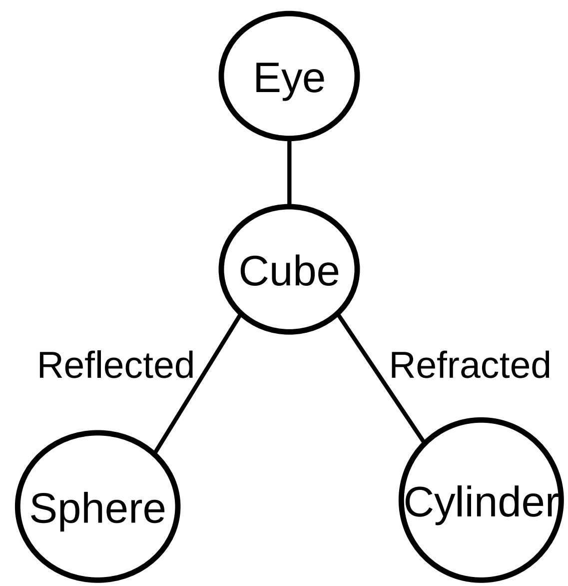
Ray Tracing Pipeline (7)



Recursive Ray Tracing



- **Searching recursively for paths to light sources**
  - Interaction of light & material at intersections
  - Recursively trace new rays in reflection, refraction, and light direction



Ray Tracing Algorithm

- **Trace(ray)**
  - Search the next intersection point (hit, material)
  - Return Shade(ray, hit, material)
- **Shade(ray, hit, material)**
  - For each light source
    - if ShadowTrace(ray to light source, distance to light)
  - Calculate reflected radiance (i.e. Phong)
  - Adding to the reflected radiance
  - If mirroring material
    - Calculate radiance in reflected direction: Trace(R(ray, hit))
    - Adding mirroring part to the reflected radiance
  - Same for transmission
  - Return reflected radiance
- **ShadowTrace(ray, dist)**
  - Return false, if intersection with distance < dist has been found
  - Can be changed to handle transparent objects as well
    - But not with refraction

Shading

- **Intersection point determines primary ray's "color"**
  - Diffuse object: color at intersection point
    - No variation with viewing angle: diffuse (Lambertian)
  - Perfect reflection/refraction (mirror, glass)
    - Only one outgoing direction → Trace one secondary ray
  - Non-Lambertian Reflectance
    - Appearance depends on illumination and viewing direction
    - Local Bi-directional Reflectance Distribution Function (BRDF)
- **Illumination**
  - Point/directional light sources
  - Area light sources
    - Approximate with multiple samples / shadow rays
  - Indirect illumination
    - See Realistic Image Synthesis (RIS) course in next semester
- **More details later**

Common Approximations

- **Usually RGB color model instead of full spectrum**
- **Finite # of point lights instead of full indirect light**
- **Approximate material reflectance properties**
  - Ambient: constant, non-directional background light
  - Diffuse: light reflected uniformly in all directions
  - Specular: perfect reflection, refraction
- **Reflection models are often empirical**

Ray Tracing Features

- **Incorporates into a single framework**
  - Hidden surface removal
    - Front to back traversal
    - Early termination once first hit point is found
  - Shadow computation
    - Shadow rays/ shadow feelers are traced between a point on a surface and a light sources
  - Exact simulation of some light paths
    - Reflection (reflected rays at a mirror surface)
    - Refraction (refracted rays at a transparent surface, Snell's law)
- **Limitations**
  - Many reflections (exponential increase in number of rays)
  - Indirect illumination requires many rays to sample all incoming directions
  - Easily gets inefficient for full global illumination computations
  - Solved with Path Tracing (→ later)

Ray Tracing Can…

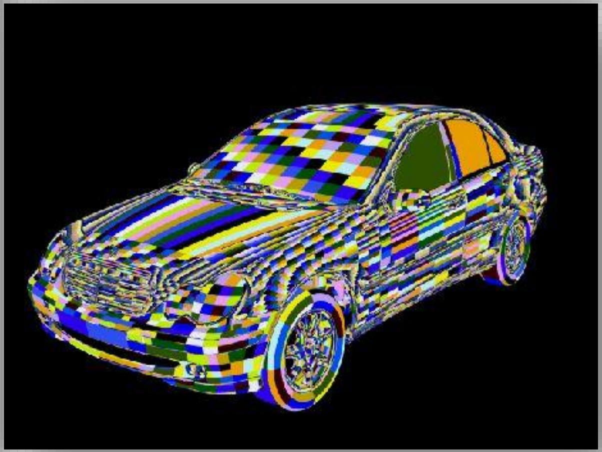
- **Produce Realistic Images**
  - By simulating light transport

What is Possible?

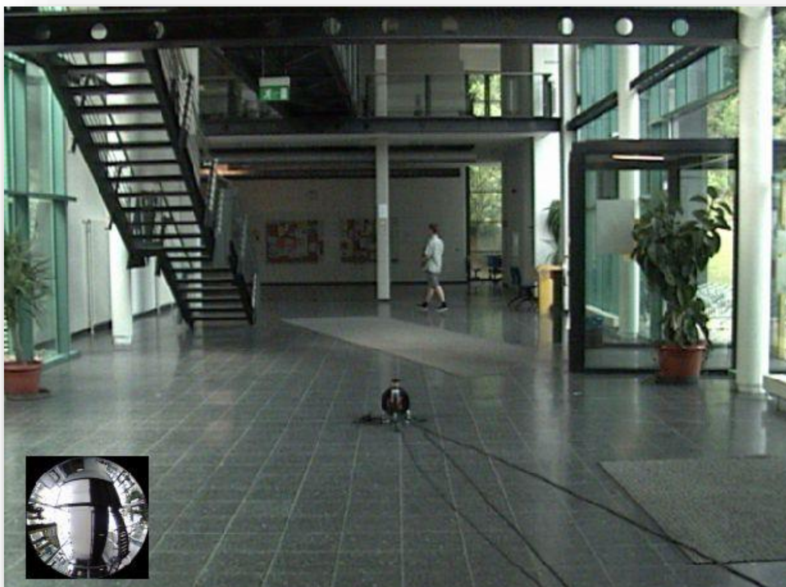
- **Models Physics of Global Light Transport**
  - Dependable, physically-correct visualization

VW Visualization Center

Realistic Visualization: CAD



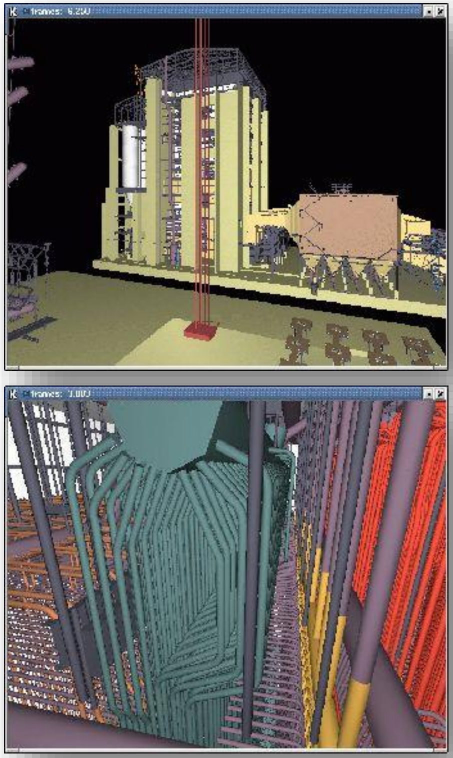
Realistic Visualization: VR/AR



Lighting Simulation

What is Possible?

- **Huge Models**
  - Logarithmic scaling in scene size

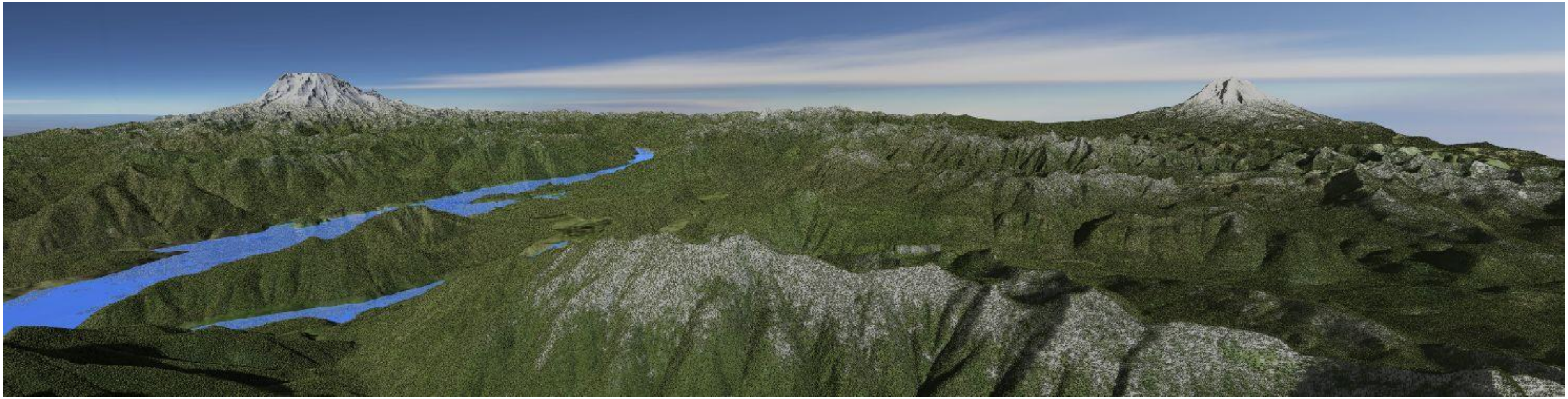


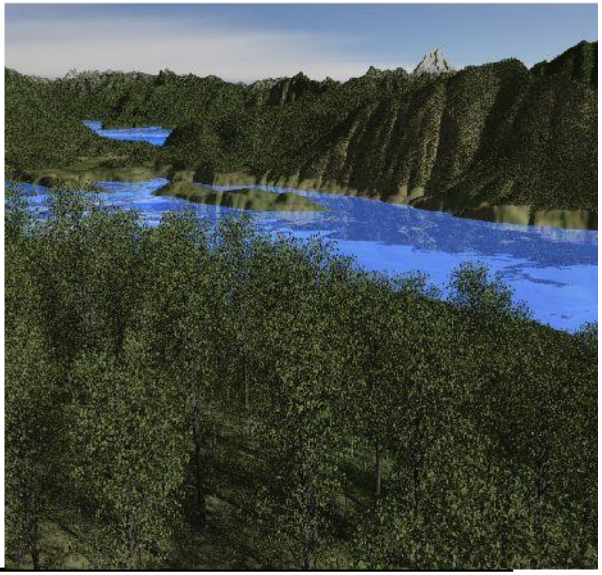
12.5 Million Triangles

~1 Billion Triangles

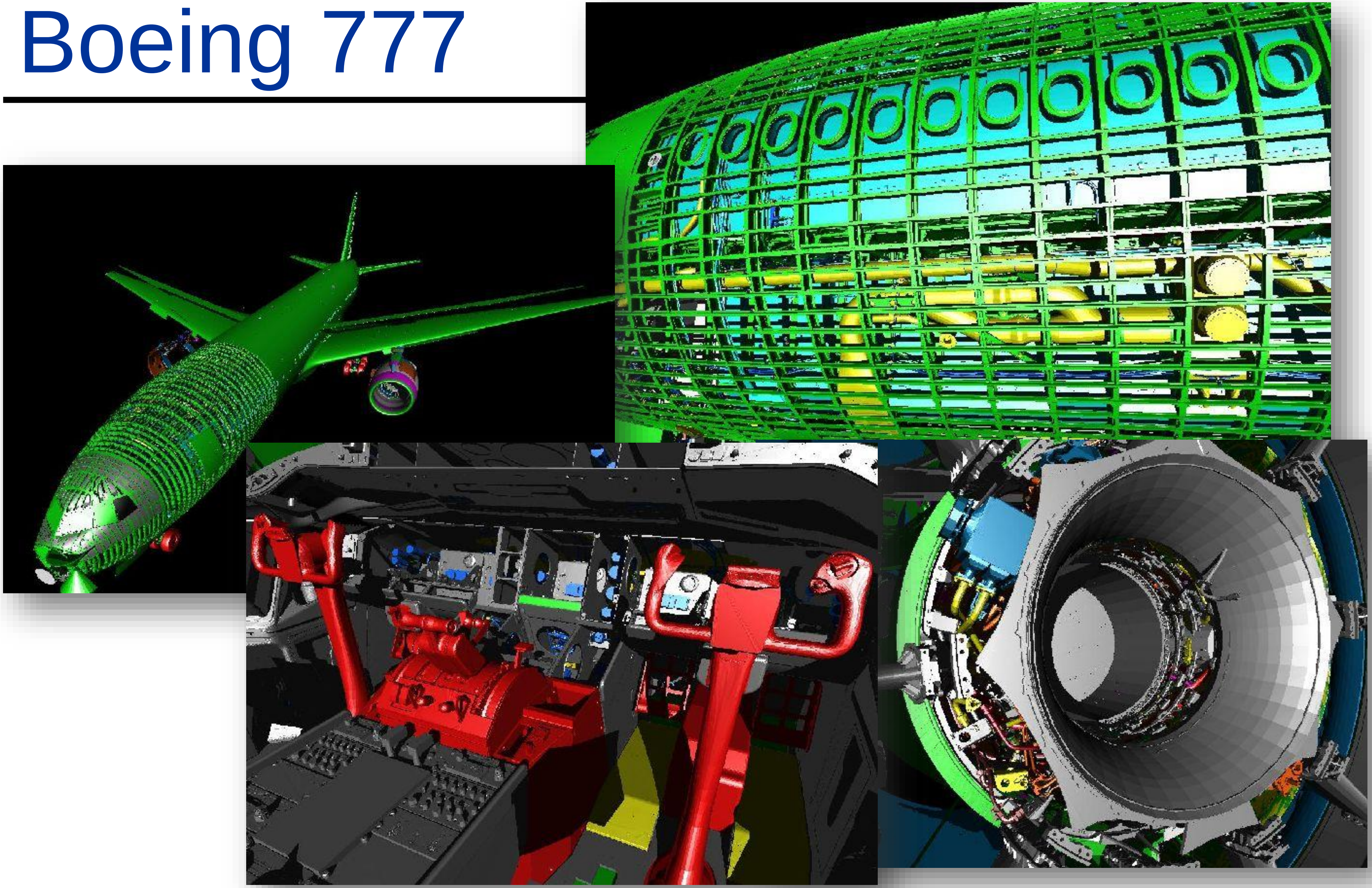
Outdoor Environments

- **90 x 10^12 (trillion) triangles**



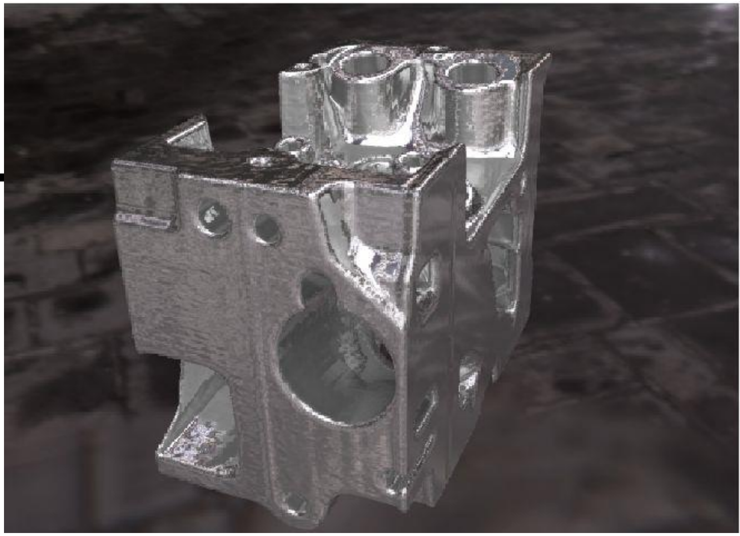


Boeing 777

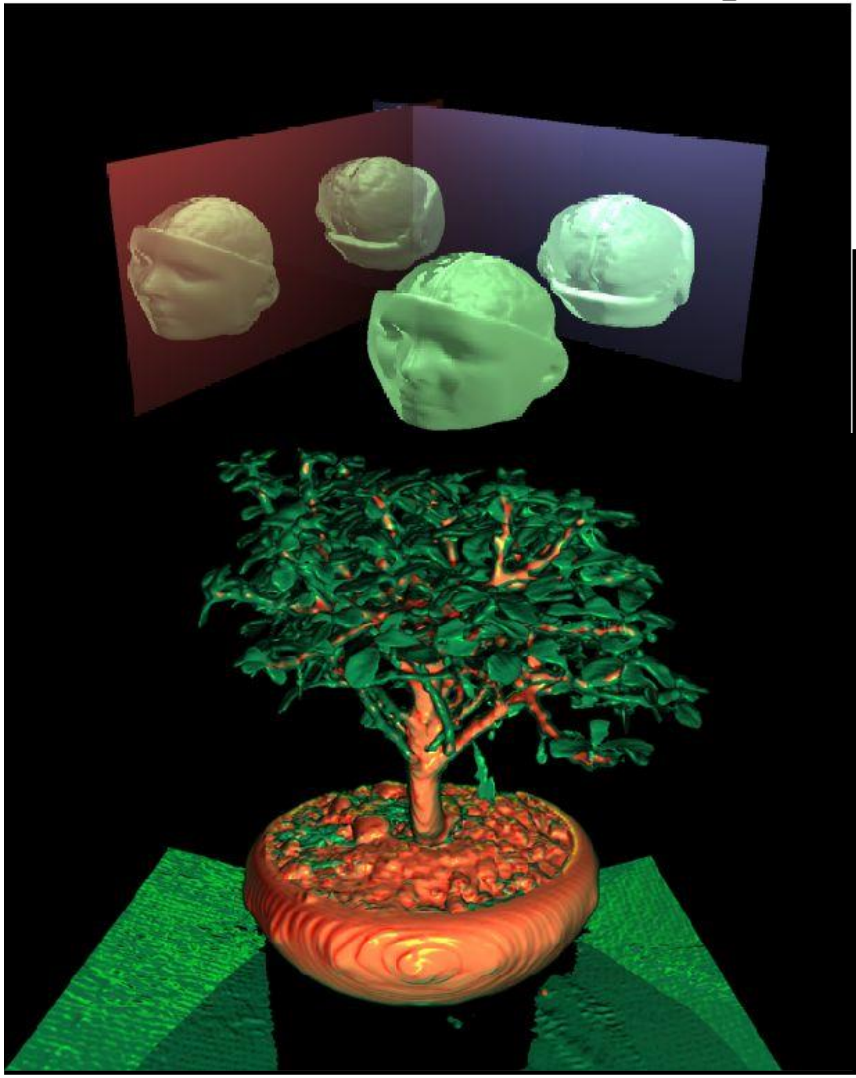


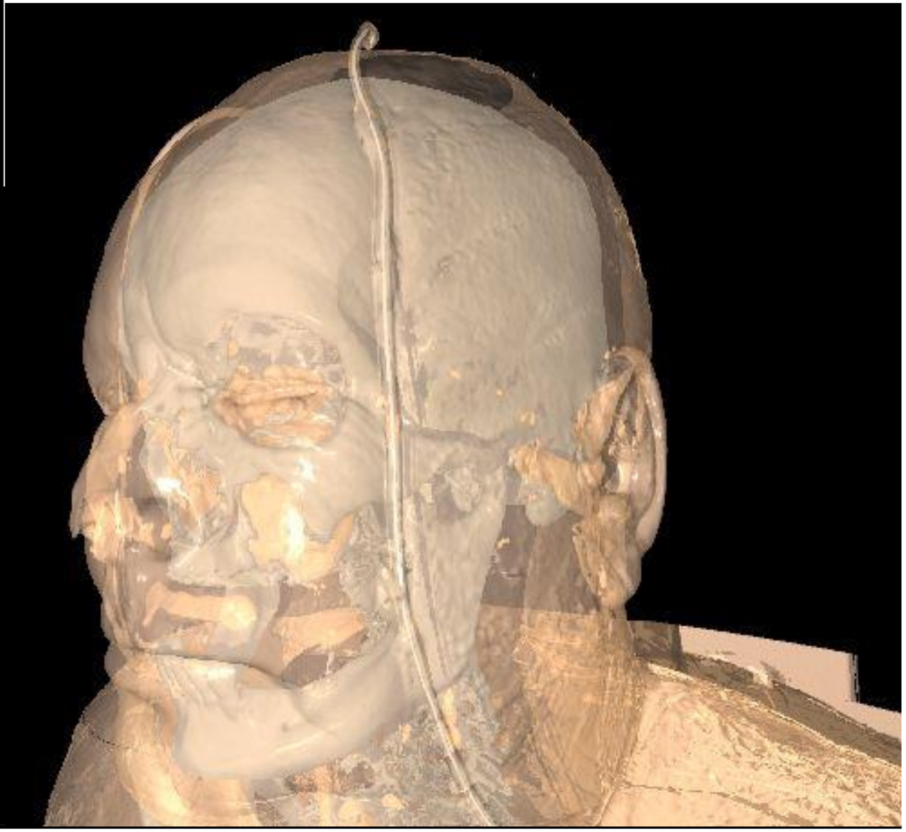
Boeing 777: ~350 million individual polygons, ~30 GB on disk

Volume Visualization



- **Iso-surface rendering**





Games?

Ray Tracing in CG

- **In the Past**
  - Only used as an off-line technique
  - Was computationally far too demanding (minutes to hours per frame)
  - Believed to not be suitable for a HW implementation
- **More Recently**
  - Interactive ray tracing on supercomputers [Parker, U. Utah‘98]
  - Interactive ray tracing on PCs [Wald‘01]
  - Distributed Real-time ray tracing on PC clusters [Wald’01]
  - RPU: First full HW implementation [Siggraph 2005]
  - Commercial tools: Embree/OSPRey (Intel/CPU), OptiX (Nvidia/GPU)
  - Complete film industry has switched to ray tracing (Monte-Carlo)
- **Own conference**
  - Symposium on Interactive RT, now High-Performance Graphics (HPG)
- **Ray tracing systems**
  - Research: PBRT (offline, physically-based, based on book, OSS), Mitsuba renderer (EPFL), imbatracer (SB), …
  - Commercial: V-Ray (Chaos Group), Corona (Render Legion), VRED (Autodesk), MentalRay/iRay (MI), …

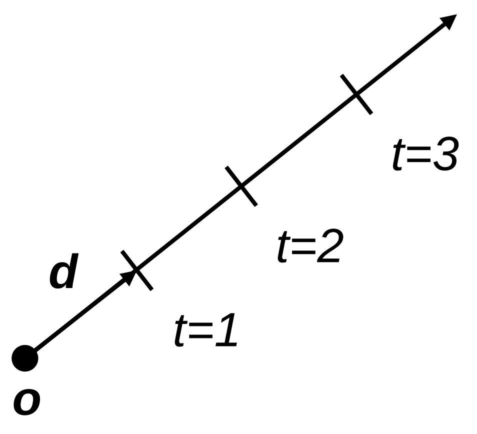
Ray Casting Outside CG

- **Tracing/Casting a ray**
  - Type of query
    - "Is there a primitive along a ray"
    - "How far is the closest primitive"
- **Other uses than rendering**
  - Volume computation
  - Sound waves tracing
  - Collision detection
  - …

RAY-PRIMITIVE INTERSECTIONS

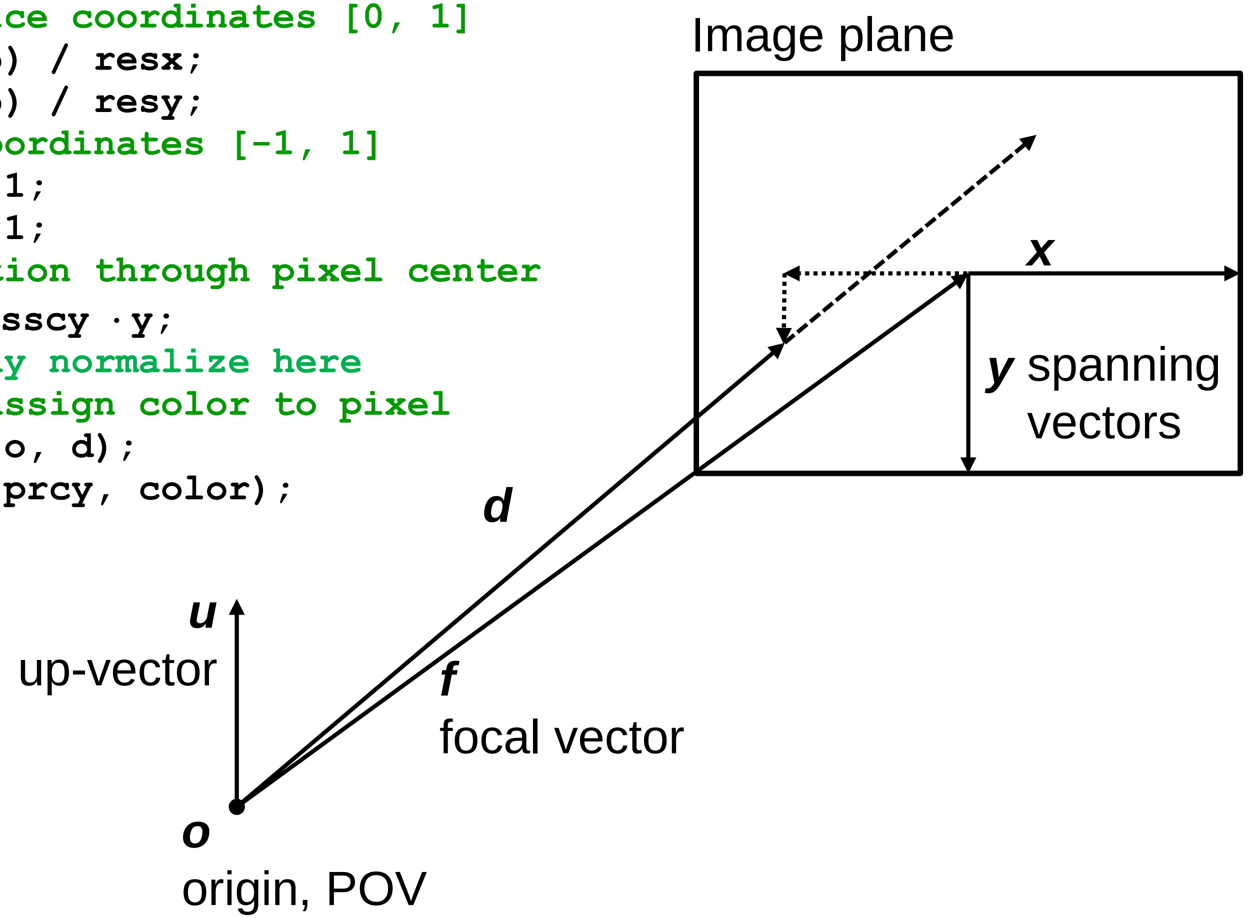
Basic Math - Ray

- **Ray parameterization**
  - $r(t) = \vec{o} + t\vec{d}$, $t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$: origin and direction
- **Ray**
  - All points on the graph of $r(t)$, with $t \in \mathbb{R}_{0+}$



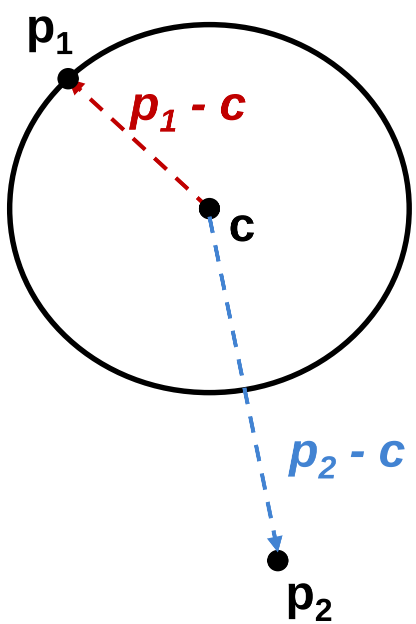
Pinhole Camera Model

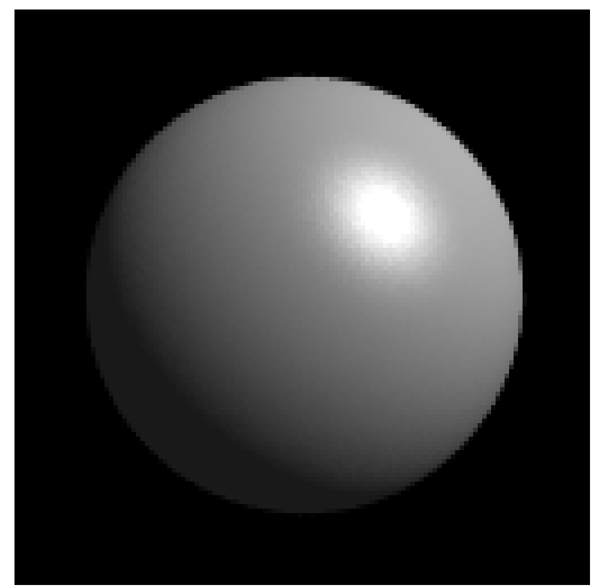
```
// For given image resolution {resx, resy}
// Loop over pixel raster coordinates [0, res-1]
for(prcx = 0; prcx < resx; prcx++)
  for(prcy = 0; prcy < resy; prcy++)
  {
    // Normalized device coordinates [0, 1]
    ndcx = (prcx + 0.5) / resx;
    ndcy = (prcy + 0.5) / resy;
    // Screen space coordinates [-1, 1]
    sscx = ndcx * 2 - 1;
    sscy = ndcy * 2 - 1;
    // Generate direction through pixel center
    d = f + sscx ·x + sscy ·y;
    d = d / |d|; // May normalize here
    // Trace ray and assign color to pixel
    color = trace_ray(o, d);
    write_pixel(prcx, prcy, color);
  }
```



Basic Math - Sphere

- **Sphere** S
  - $\vec{c} \in \mathbb{R}^3, r \in \mathbb{R}$: center and radius
  - $\forall \vec{p} \in \mathbb{R}^3: \vec{p} \in S \Leftrightarrow (\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) - r^2 = 0$
    - The distance between the points on the sphere and its center equals the radius



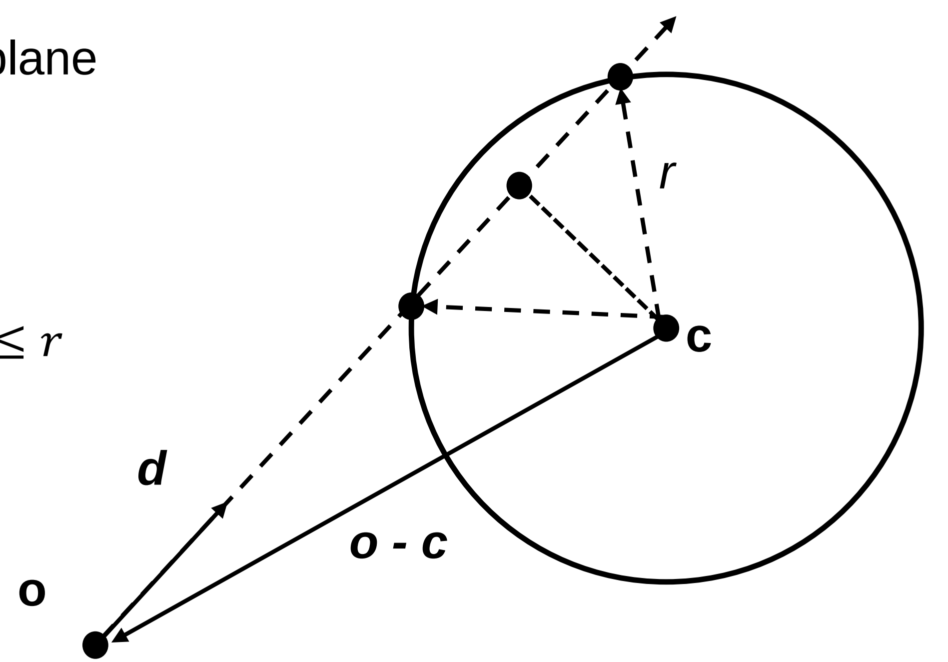


Ray-Sphere Intersection

- **Given**
  - Ray: $r(t) = \vec{o} + t\vec{d}$, t $\in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
  - Sphere: $\vec{c} \in \mathbb{R}^3, r \in \mathbb{R}$:
    - $\forall \vec{p} \in \mathbb{R}^3 : \vec{p} \in S \Leftrightarrow (\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) - r^2 = 0$
- **Find closest intersection point**
  - Algebraic approach: substitute ray equation
    - $(\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) - r^2 = 0$ with $\vec{p} = \vec{o} + t\vec{d}$
    - $t^2\vec{d} \cdot \vec{d} + 2t\vec{d} \cdot (\vec{o} - \vec{c}) + (\vec{o} - \vec{c}) \cdot (\vec{o} - \vec{c}) - r^2 = 0$
    - Solve for *t*

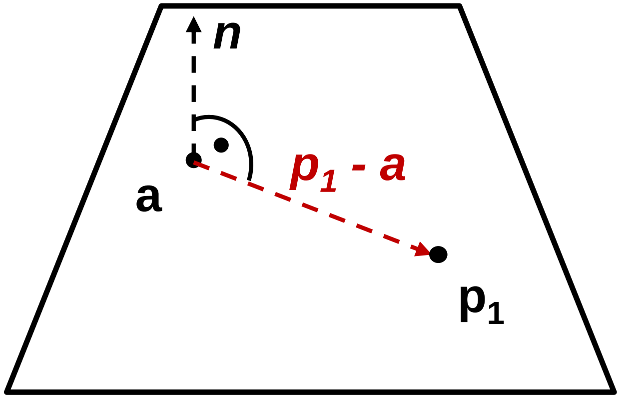
Ray-Sphere Intersection (2)

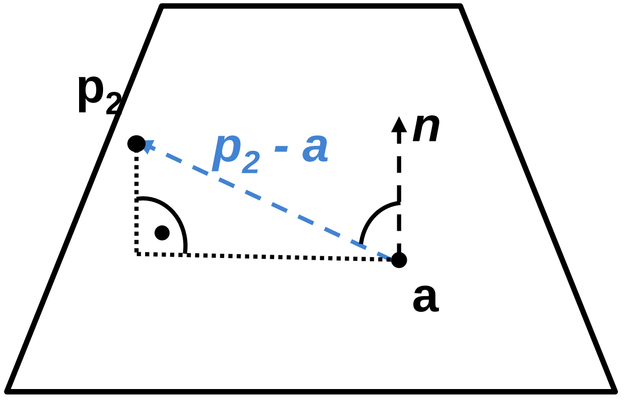
- **Given**
  - Ray: $r(t) = \vec{o} + t\vec{d}$, t $\in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
  - Sphere: $\vec{c} \in \mathbb{R}^3, r \in \mathbb{R}$:
    - $\forall \vec{p} \in \mathbb{R}^3 : \vec{p} \in S \Leftrightarrow (\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) - r^2 = 0$
- **Find closest intersection point**
  - Geometric approach
    - Ray and center span a plane
    - Solve in 2D
    - Compute $|\vec{b} - \vec{o}|, |\vec{b} - \vec{c}|$
      - $\measuredangle OBC = 90°$
    - Intersection(s) if $|\vec{b} - \vec{c}| \leq r$
  - Be aware of floating point issues if o is far from sphere



Basic Math - Plane

- **Plane** P
  - $\vec{n}, \vec{a} \in \mathbb{R}^3$: normal and point in P
  - $\forall \vec{p} \in \mathbb{R}^3 : \vec{p} \in P \Leftrightarrow (\vec{p} - \vec{a}) \cdot \vec{n} = 0$
    - The difference vector between any two points on the plane is either 0 or orthogonal to the plane's normal





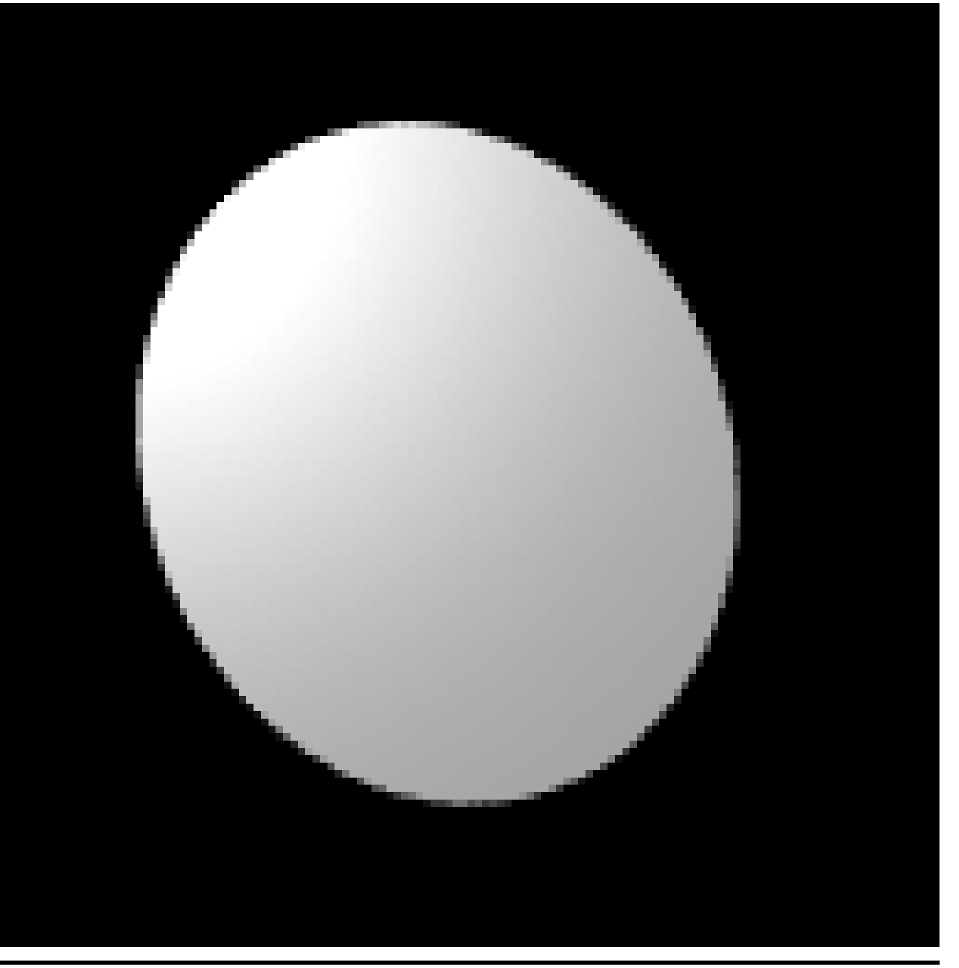
Ray-Plane Intersection

- **Given**
  - Ray: $r(t) = \vec{o} + t\vec{d}$, t $\in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
  - Plane: $\vec{n}, \vec{a} \in \mathbb{R}^3$: normal and point in P
- **Compute intersection point**
  - Plane equation: $\vec{p} \in P \Leftrightarrow (\vec{p} - \vec{a}) \cdot \vec{n} = 0$

    $\Leftrightarrow \quad \vec{p} \cdot \vec{n} - D = 0, \; with \;\; D = \vec{a} \cdot \vec{n}$
  - Substitute ray parameterization: $(\vec{o} + t\vec{d}) \cdot \vec{n} - D = 0$
  - Solve for t
    - 0,1 or infinitely many solutions

Ray-Disc Intersection

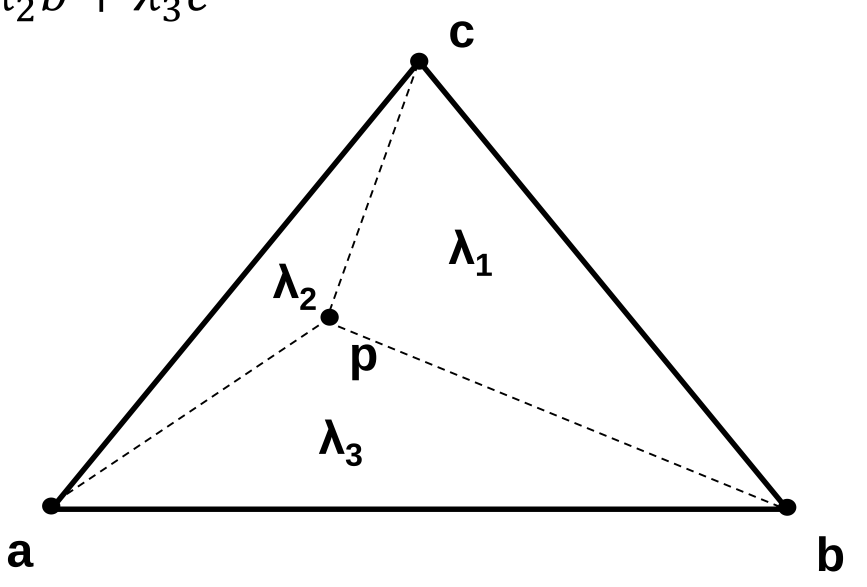
- **Intersect ray with plane**
- **Discard intersection if $||p - a|| > r$**



Basic Math - Triangle

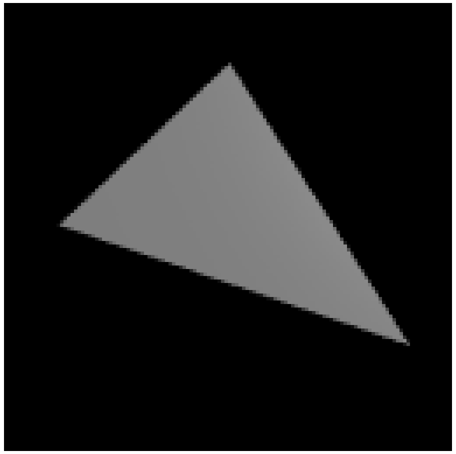
- **Triangle** T
  - $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$: vertices
  - Affine combinations of $\vec{a}, \vec{b}, \vec{c} \rightarrow$ points in the plane
    - Non-negative coefficients that sum up to 1 → points in the triangle
  - $\forall \vec{p} \in \mathbb{R}^3 : \vec{p} \in T \Leftrightarrow \exists \lambda_{1,2,3} \in \mathbb{R}_{0+},\ \lambda_1 + \lambda_2 + \lambda_3 = 1\ and$
    $$\vec{p} = \lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \vec{c}$$

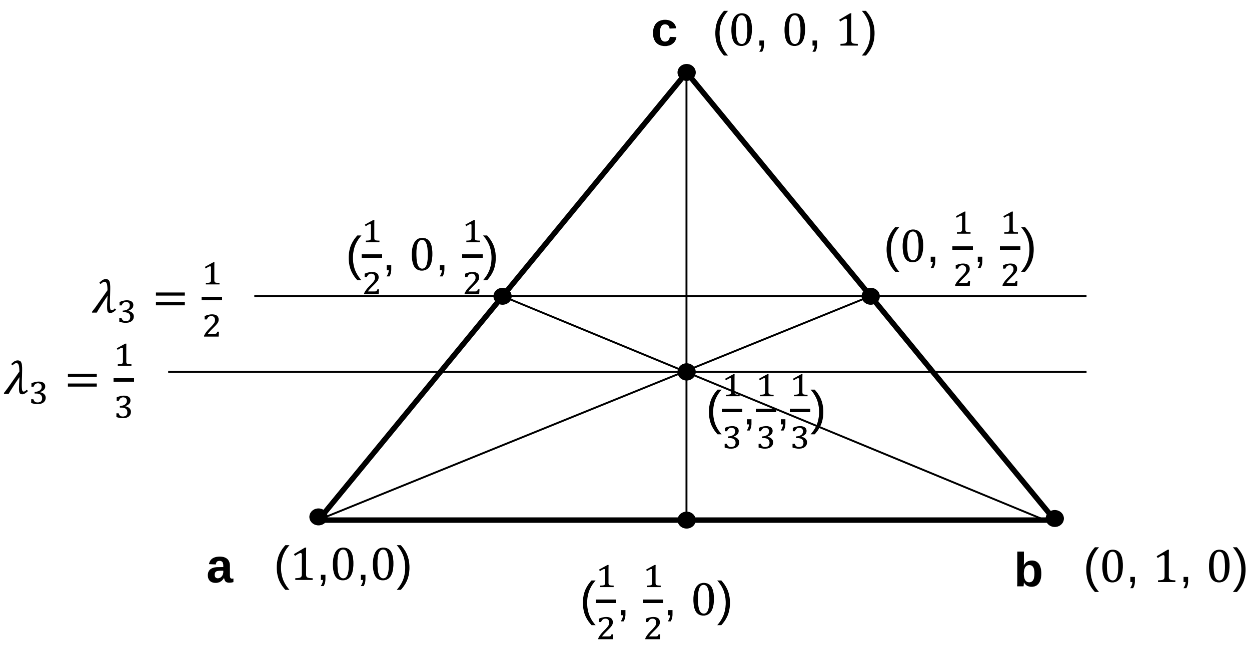
- **Barycentric coordinates**
  - $\lambda_{1,2,3}$
  - $\lambda_1 = S_{pbc} / S_{abc}$
  - S: signed area of triangles



Barycentric Coordinates

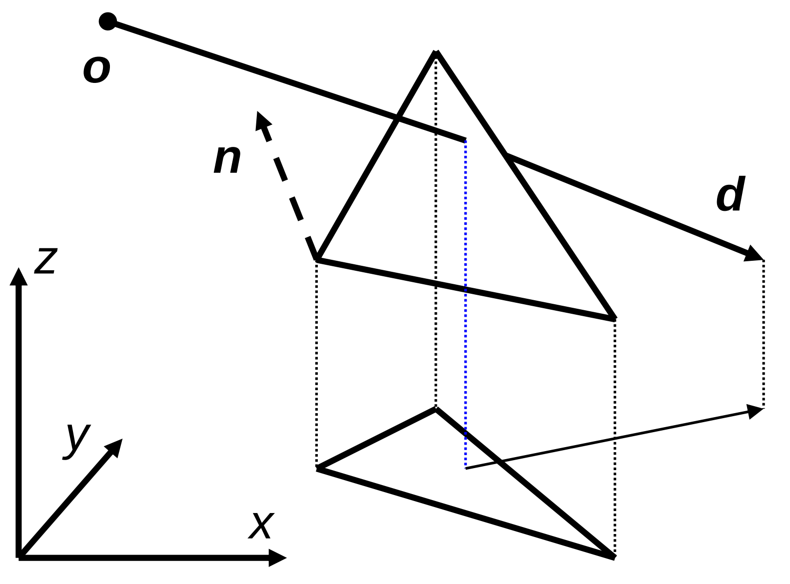
- **Triangle** T
  - $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$: vertices
  - $\lambda_{1,2,3}$: barycentric coordinates
  - $\lambda_1 + \lambda_2 + \lambda_3 = 1$
  - $\lambda_1 = S_{pbc}/S_{abc}$, etc.





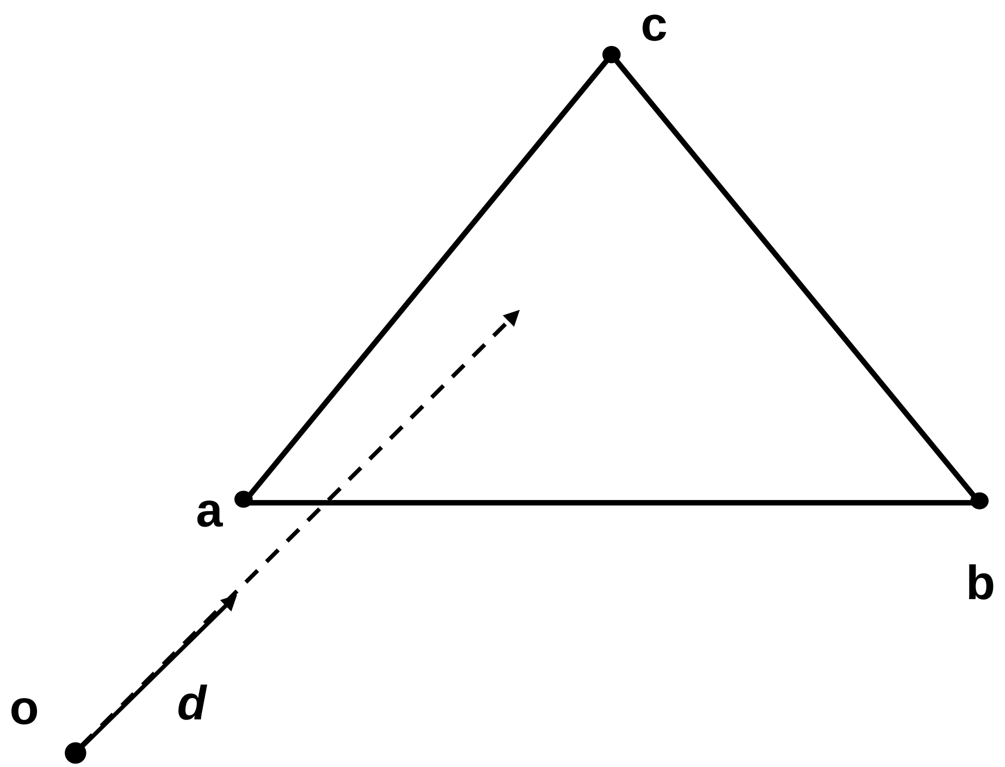
Triangle Intersection: Plane-Based

- **Compute intersection with triangle plane**
- **Compute barycentric coordinates**
  - Signed areas of subtriangles
  - Can be done in 2D, after "projection" onto major plane, depending on largest normal vector component
- **Test for positive BCs**



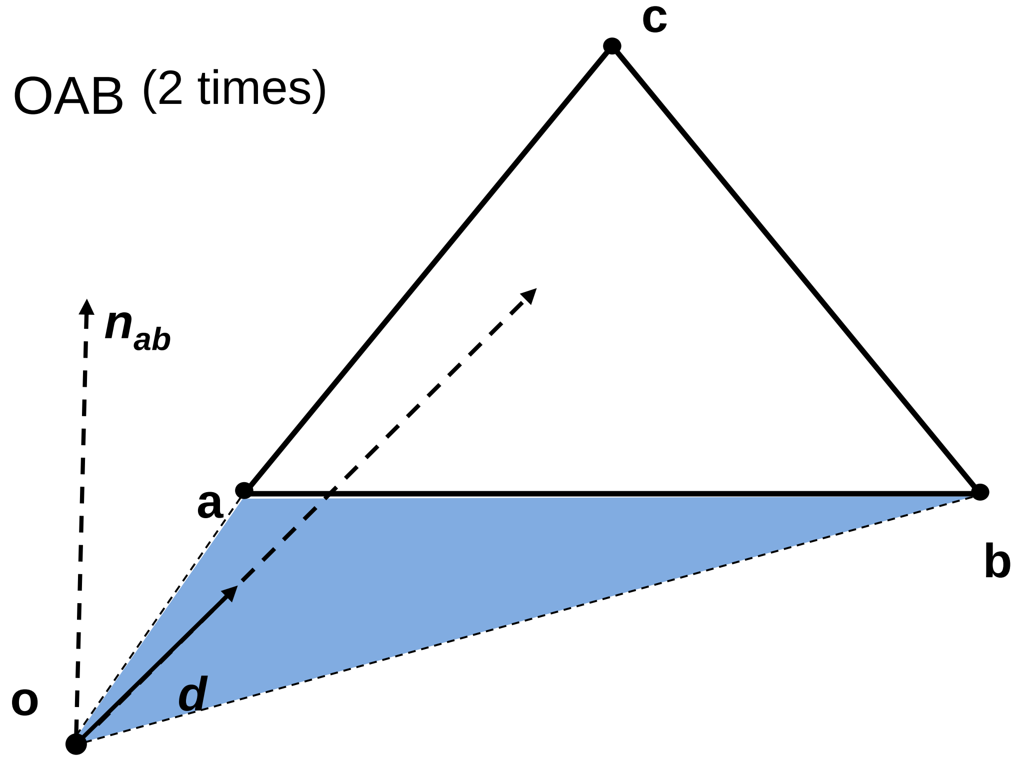
Triangle Intersection Edge-Based (1)

- **3D linear function across triangle (3D edge functions)**
  - Ray: $\vec{o} + t\vec{d}$, t $\in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
  - Triangle: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$



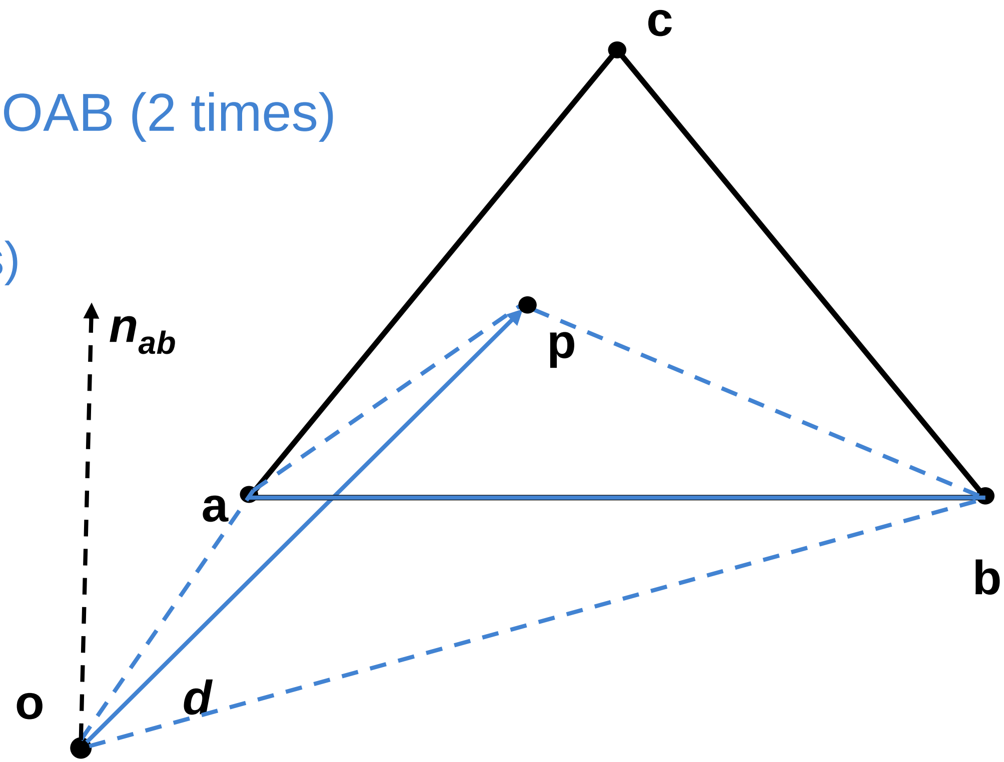
Triangle Intersection Edge-Based (2)

- **3D linear function across triangle (3D edge functions)**
  - Ray: $\vec{o} + t\vec{d}$, $t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
  - Triangle: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$
  - $\overrightarrow{n_{ab}} = \left(\vec{b} - \vec{o}\right) \times (\overrightarrow{a} - \vec{o})$
  - $|\overrightarrow{n_{ab}}|$ is the signed area of OAB (2 times)



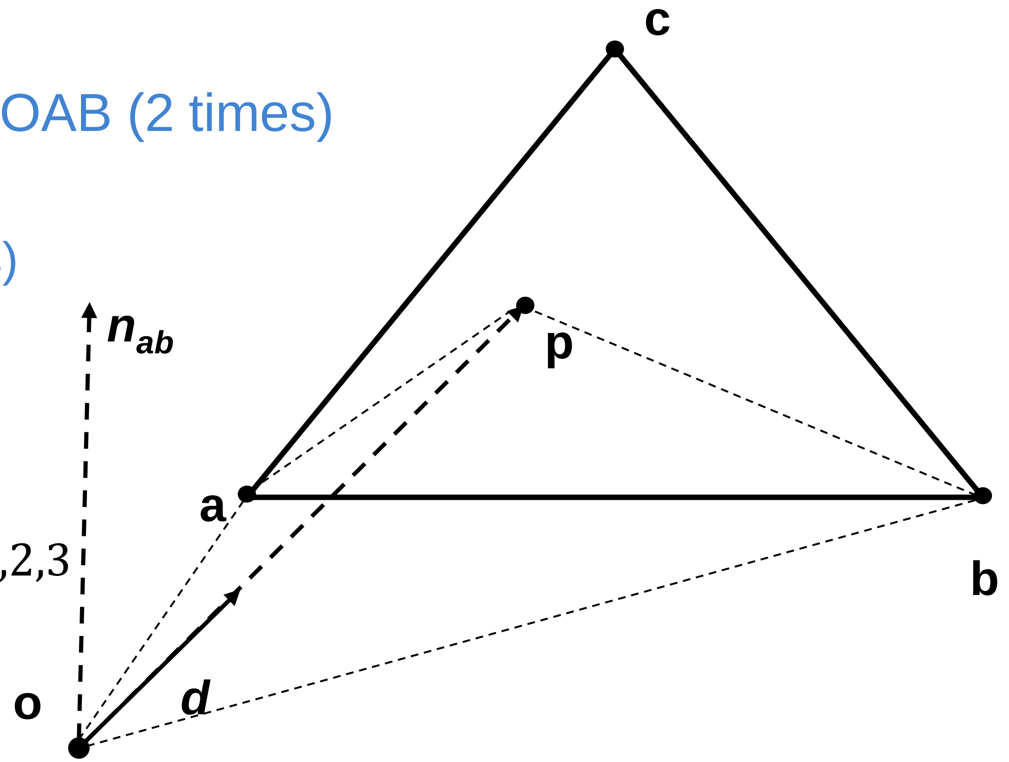
Triangle Intersection Edge-Based (3)

- **3D linear function across triangle (3D edge functions)**
  - Ray: $\vec{o} + t\vec{d}$,   t $\in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
  - Triangle: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$
  - $\overrightarrow{n_{ab}} = (\vec{b} - \vec{o}) \times (\overrightarrow{a} - \vec{o})$
  - $|\overrightarrow{n_{ab}}|$ is the signed area of OAB (2 times)
  - $\lambda_3^*(t) = \overrightarrow{n_{ab}} \cdot t\vec{d}$
    - Volume of OABP (6 times)
    - For $t = t_{hit}$



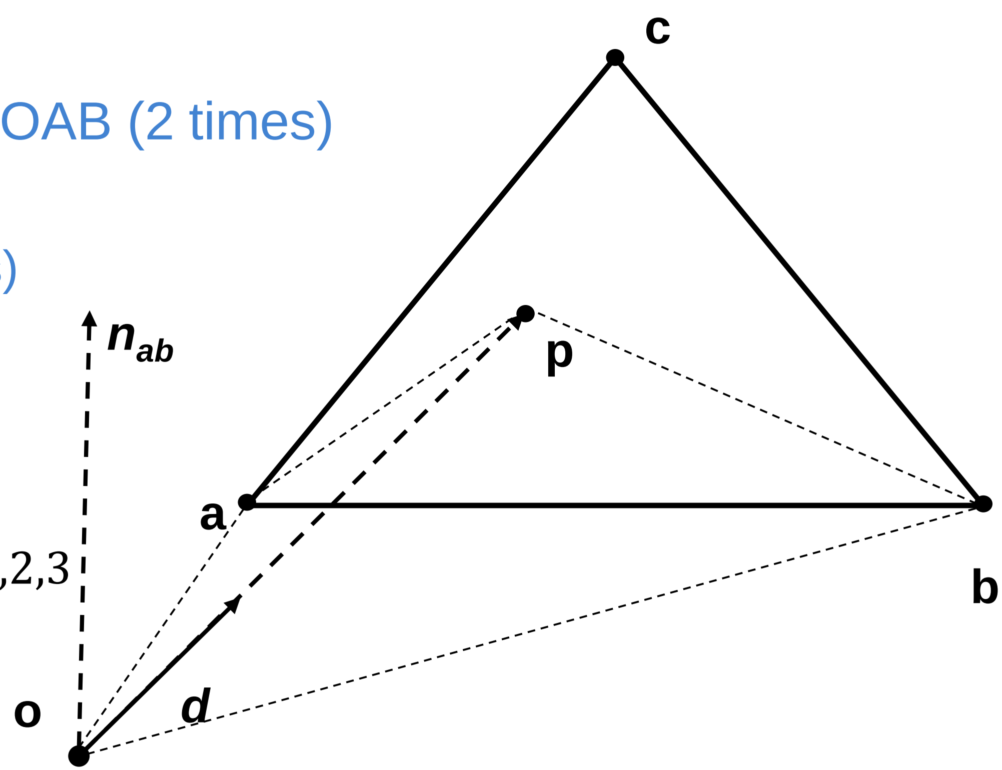
Triangle Intersection Edge-Based (4)

- **3D linear function across triangle (3D edge functions)**
  - Ray: $\vec{o} + t\vec{d}$, t ∈ $\mathbb{R}$; $\vec{o}, \vec{d} \in \mathbb{R}^3$
  - Triangle: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$
  - $\overrightarrow{n_{ab}} = (\vec{b} - \vec{o}) \times (\vec{a} - \vec{o})$
  - $|\overrightarrow{n_{ab}}|$ is the signed area of OAB (2 times)
  - $\lambda_3^*(t) = \overrightarrow{n_{ab}} \cdot t\vec{d}$
    - Volume of OABP (6 times)
    - For $t = t_{hit}$
  - $\lambda_{1,2}^*(t) = \overrightarrow{n_{bc,ac}} \cdot t\vec{d}$
  - Normalize
    - $\lambda_i = \frac{\lambda_i^*(t)}{\lambda_1^*(t)+\lambda_2^*(t)+\lambda_3^*(t)}, i = 1,2,3$
    - Length of $t\vec{d}$ cancels out



Triangle Intersection Edge-Based (5)

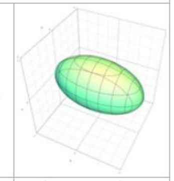
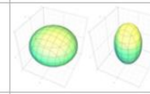
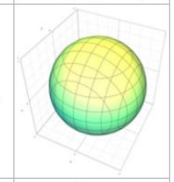
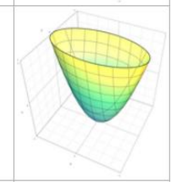
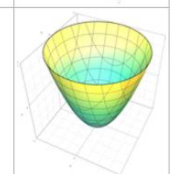
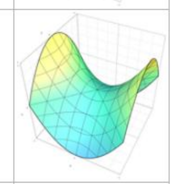
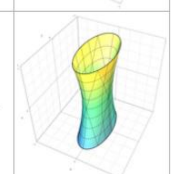
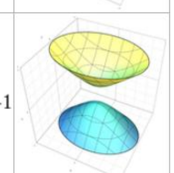
- **3D linear function across triangle (3D edge functions)**
  - Ray: $\vec{o} + t\vec{d}$, t ∈ $\mathbb{R}$; $\vec{o}, \vec{d} \in \mathbb{R}^3$
  - Triangle: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$
  - $\overrightarrow{n_{ab}} = (\vec{b} - \vec{o}) \times (\vec{a} - \vec{o})$
  - $|\overrightarrow{n_{ab}}|$ is the signed area of OAB (2 times)
  - $\lambda_3^*(t) = \overrightarrow{n_{ab}} \cdot t\vec{d}$
    - Volume of OABP (6 times)
    - For $t = t_{hit}$
  - $\lambda_{1,2}^*(t) = \overrightarrow{n_{bc,ac}} \cdot t\vec{d}$
  - Normalize
    - $\lambda_i = \frac{\lambda_i^*(t)}{\lambda_1^*(t)+\lambda_2^*(t)+\lambda_3^*(t)}, i = 1,2,3$

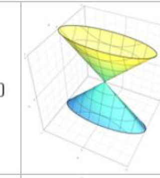
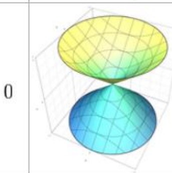
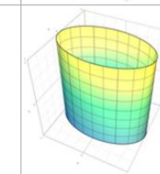
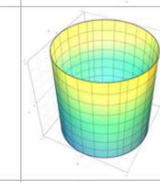
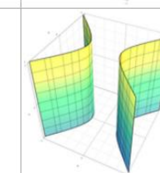
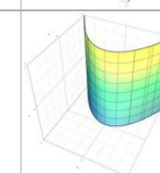


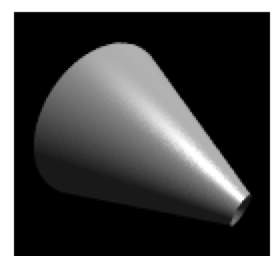
- **For positive BCs**
  - Compute $\vec{p} = \lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3\vec{c}$

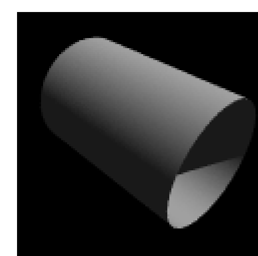
Quadrics

- **Implicit**
  - f(x, y, z) = v
- **Ray equation**
  - x = xo + t xd
  - y = yo + t yd
  - z = zo + t zd
- **Solve for t**
- **Assignment**

Non-degenerate real quadric surfaces		
Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	
Spheroid (special case of ellipsoid)	$\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{b^2}=1$	
Sphere (special case of spheroid)	$\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{a^2}=1$	
Elliptic paraboloid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-z=0$	
Circular paraboloid(special case of elliptic paraboloid)	$\frac{x^2}{a^2}+\frac{y^2}{a^2}-z=0$	
Hyperbolic paraboloid	$\frac{x^2}{a^2}-\frac{y^2}{b^2}-z=0$	
Hyperboloid of one sheet	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	
Hyperboloid of two sheets	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$	

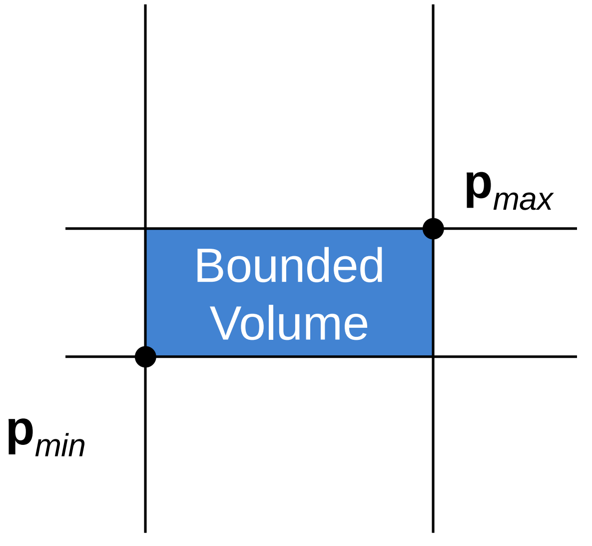
Degenerate quadric surfaces		
Cone	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$	
Circular Cone (special case of cone)	$\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{b^2}=0$	
Elliptic cylinder	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	
Circular cylinder (special case of elliptic cylinder)	$\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$	
Hyperbolic cylinder	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	
Parabolic cylinder	$x^2+2ay=0$	

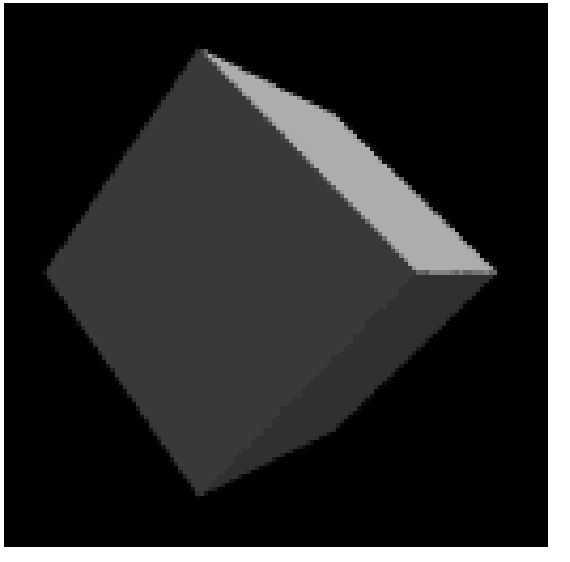




Axis Aligned Bounding Box

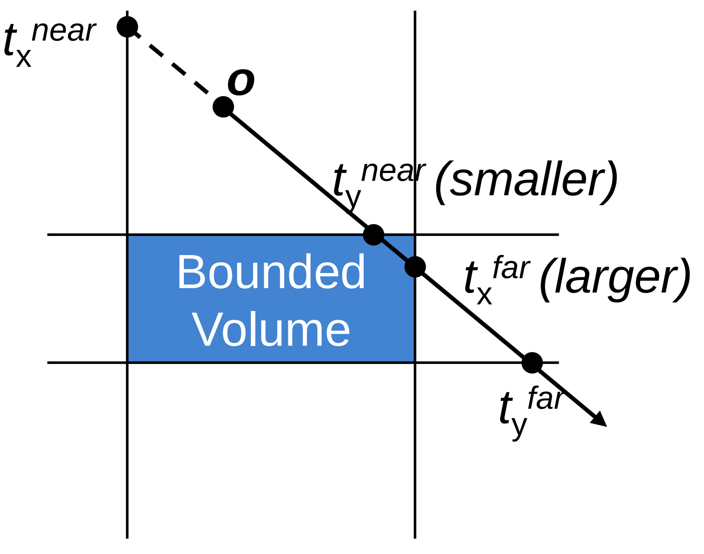
- **Given**
  - Ray: $\vec{o} + t\vec{d}$, $t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
  - Axis aligned bounding box (AABB): $\overrightarrow{p_{min}}, \overrightarrow{p_{max}} \in \mathbb{R}^3$

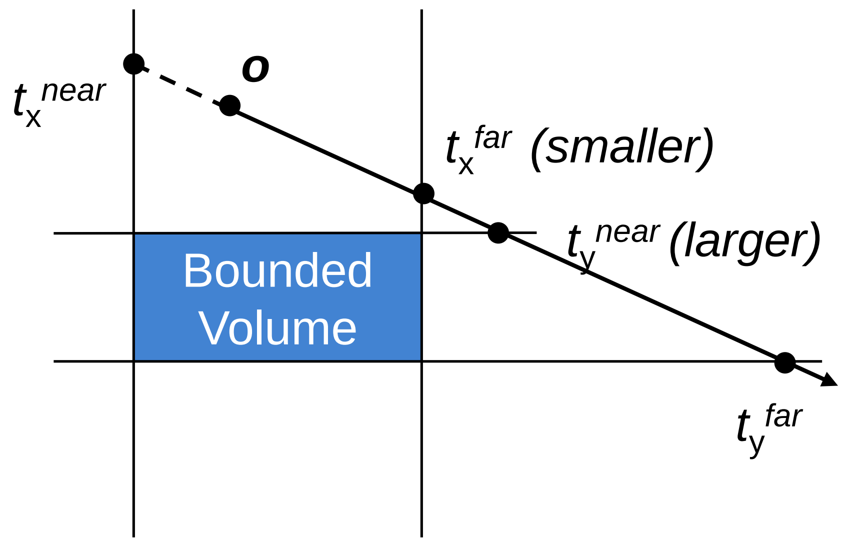




Ray-Box Intersection

- **Given**
  - Ray: $\vec{o} + t\vec{d}$, t $\in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
  - Axis aligned bounding box (AABB): $\overrightarrow{p_{min}}, \overrightarrow{p_{max}} \in \mathbb{R}^3$
- **"Slabs test" for ray-box intersection**
  - Ray enters the box in all dimensions before exiting in any
  - $\max(\{t_i^{near} | i = x, y, z\}) < \min(\{t_i^{far} | i = x, y, z\})$



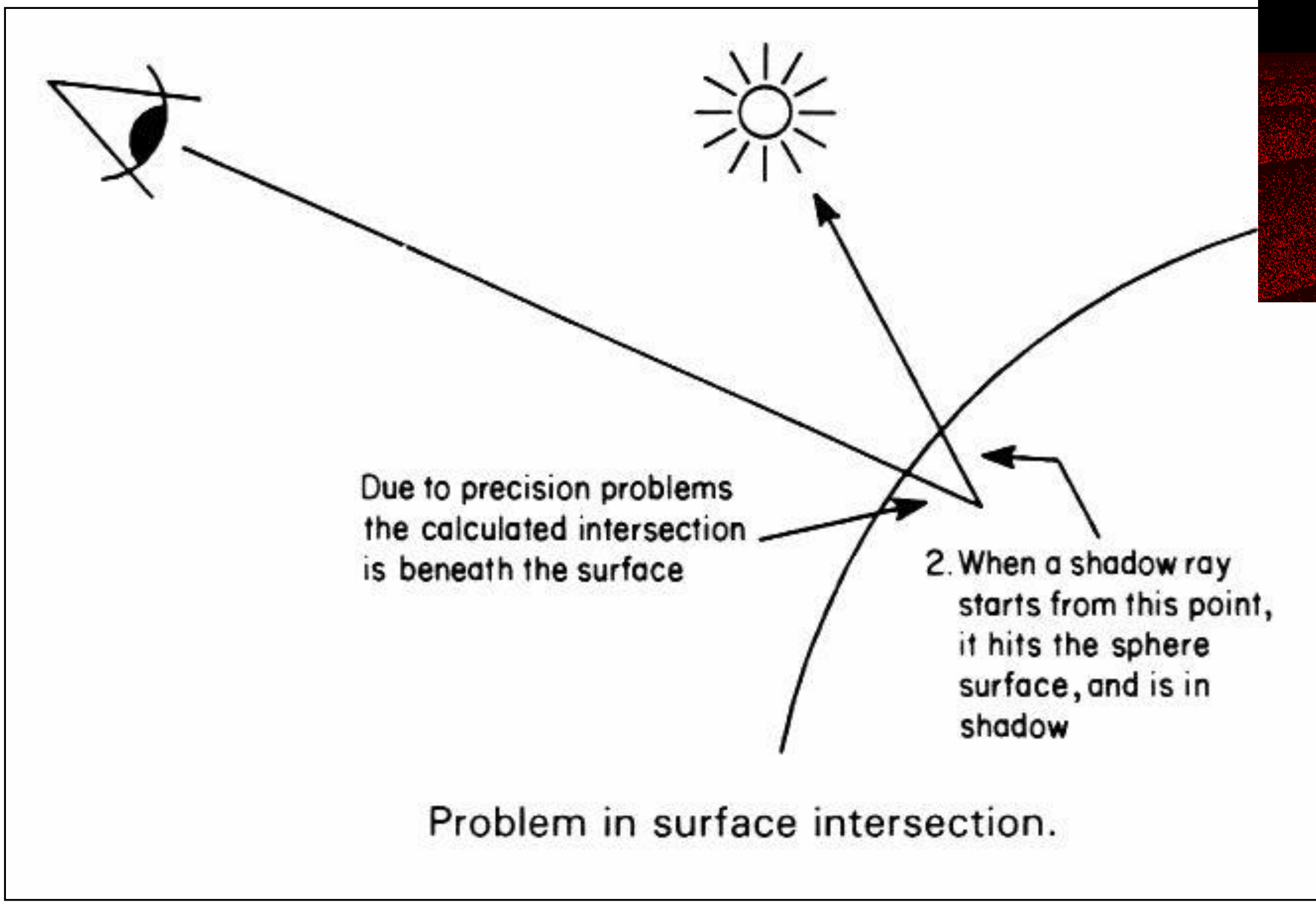


History of Intersection Algorithms

- **Ray-geometry intersection algorithms**
  - Polygons: [Appel ’68]
  - Quadrics, CSG: [Goldstein & Nagel ’71]
  - Recursive Ray Tracing: [Whitted ’79]
  - Tori: [Roth ’82]
  - Bicubic patches: [Whitted ’80, Kajiya ’82]
  - Algebraic surfaces: [Hanrahan ’82]
  - Swept surfaces: [Kajiya ’83, van Wijk ’84]
  - Fractals: [Kajiya ’83]
  - Deformations: [Barr ’86]
  - NURBS: [Stürzlinger ’98]
  - Subdivision surfaces: [Kobbelt et al ’98]

Precision Problems

- **Cause of „surface acne“**



Problem in surface intersection.