Computer Graphics

- Clipping -

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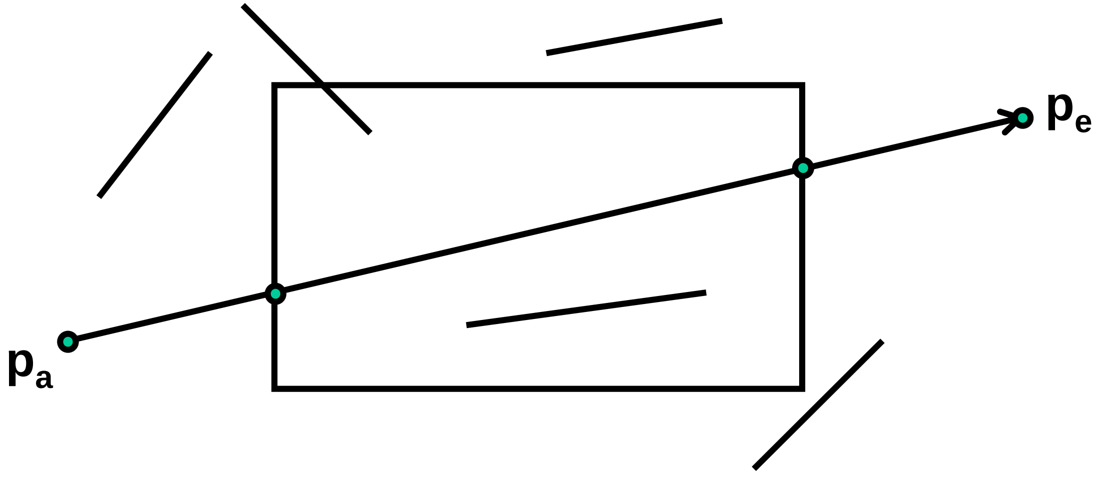
Clipping

- **Motivation**
  - Projected primitive might fall (partially) outside of display area
    - E.g. if standing inside a building
  - Eliminate non-visible geometry early in the pipeline to process visible parts only
  - Happens after transformation from 3D to 2D
  - Must cut off parts outside the window
    - Cannot draw outside of window (e.g. plotter)
    - Outside geometry might not be representable (e.g. in fixed point)
  - Must maintain information properly
    - Drawing the clipped geometry should give the correct results: e.g. correct interpolation of colors at triangle vertices when one is clipped
    - Type of geometry might change
      - Cutting off a vertex of a triangle produces a quadrilateral
      - Might need to be split into triangle again
    - Polygons must remain closed after clipping

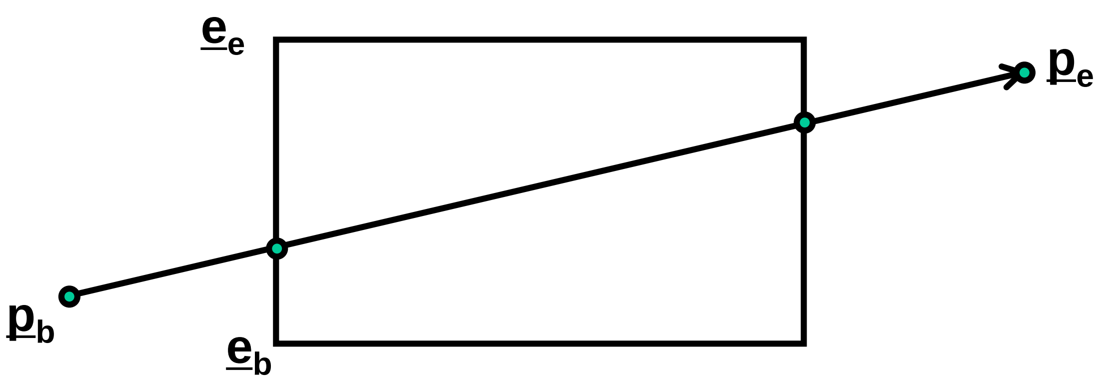
Line Clipping

- **Definition of clipping**
  - Cut off parts of objects which lie outside/inside of a defined region
  - Often clip against viewport (2D) or canonical view-volume (3D)
- **Let`s focus first on lines only**



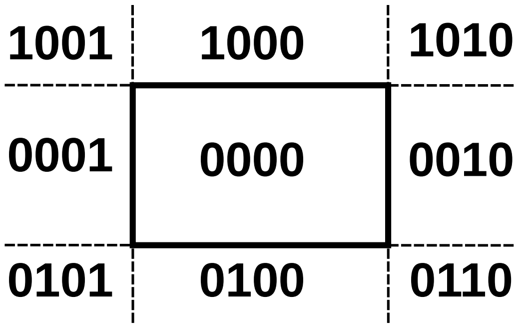
Brute-Force Method

- **Brute-force line clipping at the viewport**
  - If both end points p_b and p_e are inside viewport
    - Accept the whole line
  - Otherwise, clip the line at *each edge*
    - $\mathrm{p}_{\mathrm{intersection}} = p_b + t_{line}(p_e - p_b) = e_b + t_{edge}(e_e - e_b)$
    - Solve for t_{line} and t_{edge}
      - Intersection within segment if both $0 \leq t_{line}, t_{edge} \leq 1$
    - Replace suitable end points for the line by the intersection point
  - Unnecessarily test many cases that are irrelevant



Cohen-Sutherland (1974)

- **Advantage: divide and conquer**
  - Efficient trivial accept and trivial reject
  - Non-trivial case: divide and test
- **Outcodes of points**
  - Bit encoding (**o**ut**c**ode, OC)
    - Each viewport edge defines a half space
    - Set bit if vertex is outside w.r.t. that edge



Bit order: *top, bottom, right, left*

Viewport (x_{min}, y_{min}, x_{max}, y_{max})

- **Trivial cases**
  - Trivial accept: both are in viewport
    - ($OC(p_b)$ **OR** $OC(p_e)$) == 0
  - Trivial reject: both lie outside w.r.t. *at least one common edge*
    - ($OC(p_b)$ **AND** $OC(p_e)$) $\neq 0$
  - Line has to be clipped to all edges where XOR bits are set, i.e. the points lies on different sides of that edge
    - $OC(p_b)$ **XOR** $OC(p_e)$

Cohen-Sutherland

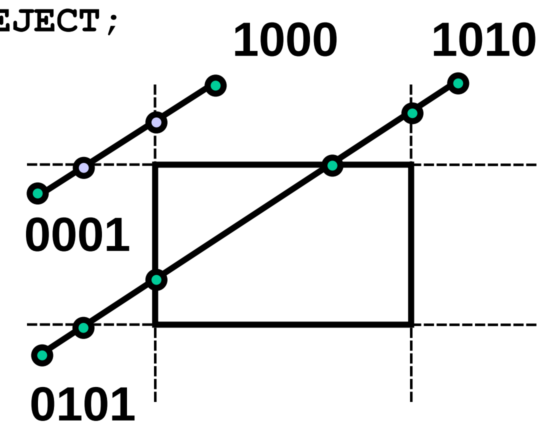
- **Clipping of line (p1, p2)**

```
oc1 = OC(p1); oc2 = OC(p2); edge = 0;
do {
  if ((oc1 AND oc2) != 0)      // trivial reject of remaining segment
    return REJECT;
  else if ((oc1 OR oc2) == 0) // trivial accept of remaining segment
    return (ACCEPT, p1, p2);
  if ((oc1 XOR oc2)[edge]) {
    if (oc1[edge])               // p1 outside
      {p1 = cut(p1, p2, edge); oc1 = OC(p1);}
    else                         // p2 outside
      {p2 = cut(p1, p2, edge); oc2 = OC(p2);}
  }
} while (++edge < 4);          // Not the most efficient solution
return ((oc1 OR oc2) == 0) ? (ACCEPT, p1, p2) : REJECT;
```

- **Intersection calculation for $x = x_{\min}$**

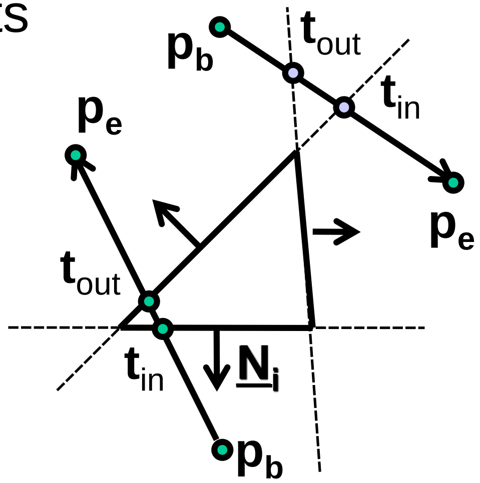
$$\frac{y - y_a}{y_e - y_a} = \frac{\mathrm{x}_{\min} - x_a}{x_e - x_a}$$

$$y = y_a + (\mathrm{x}_{\min} - x_a)\frac{y_e - y_a}{x_e - x_a}$$

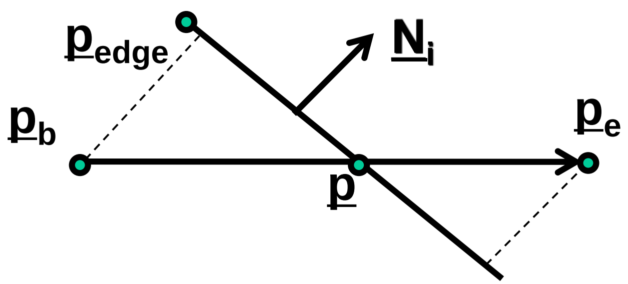


Cyrus-Beck (1978)

- **Parametric line-clipping algorithm**
  - Only convex polygons: max 2 intersection points
  - Use edge orientation
- **Idea: clipping against polygons**
  - Clip line $\mathrm{p} = p_b + t_i(p_e - p_b)$ with each edge
  - Intersection points sorted by parameter t_i
  - Select
    - t_{in}: entry point ($(p_e - p_b) \cdot N_i < 0$) with largest t_i
    - t_{out}: exit point ($(p_e - p_b) \cdot N_i > 0$) with smallest t_i
  - If $t_{out} < t_{in}$, line lies completely outside (akin to ray-box intersect.)



- **Intersection calculation**



$$\left(p - p_{edge}\right) \cdot N_i = 0$$

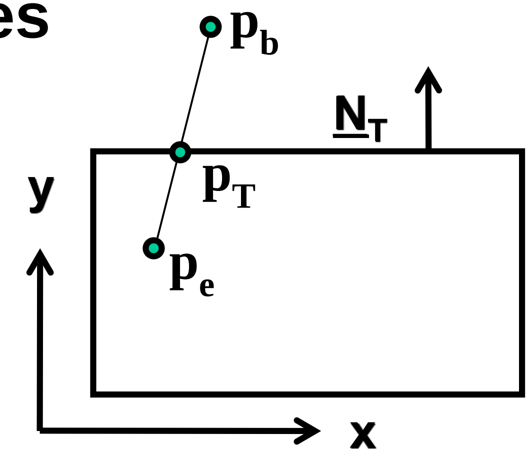
$$t_i(p_e - p_b) \cdot N_i + \left(p_b - p_{edge}\right) \cdot N_i = 0$$

$$t_i = \frac{\left(p_{edge} - p_b\right) \cdot N_i}{(p_e - p_b) \cdot N_i}$$

Liang-Barsky (1984)

- **Cyrus-Beck for axis-aligned rectangles**
  - Using window-edge coordinates (with respect to an edge T)

$$WEC_T(p) = (p - p_T) \cdot N_T$$



- **Example: top (y = y_{max})**

$$N_T = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad p_b - p_T = \begin{pmatrix} x_b - x_{\max} \\ y_b - y_{\max} \end{pmatrix}$$

$$t_T = \frac{(p_b - p_T) \cdot N_T}{(p_b - p_e) \cdot N_T} = \frac{\mathrm{WEC}_T(p_b)}{\mathrm{WEC}_T(p_b) - \mathrm{WEC}_T(p_e)} = \frac{y_b - y_{\max}}{y_b - y_e}$$

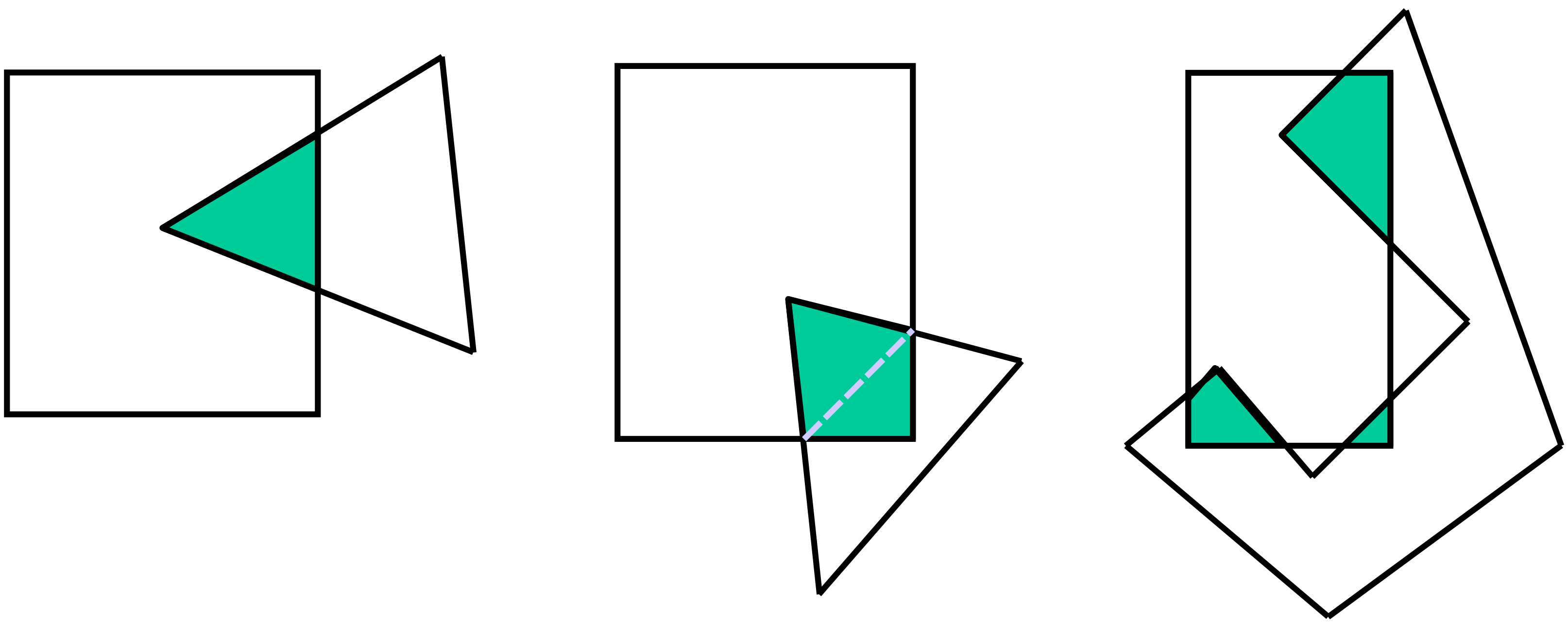
  - Window-edge coordinate (WEC): decision function for an edge
    - Directed distance to edge
      - Only sign matters, similar to Cohen-Sutherland opcode
    - Sign of the dot product determines whether the point is in or out
    - Normalization unimportant

Line Clipping - Summary

- **Cohen-Sutherland, Cyrus-Beck, and Liang-Barsky algorithms readily extend to 3D**
- **Cohen-Sutherland algorithm**
  - \+ Efficient when majority of lines can be trivially accepted / rejected
    - Very large clip rectangles: almost all lines inside
    - Very small clip rectangles: almost all lines outside
  - – Repeated clipping for remaining lines
  - – Testing for 2D/3D point coordinates
- **Cyrus-Beck (Liang-Barsky) algorithms**
  - \+ Efficient when many lines must be clipped
  - \+ Testing for 1D parameter values
  - – Testing intersections always for all clipping edges (in the Liang-Barsky trivial rejection testing possible)

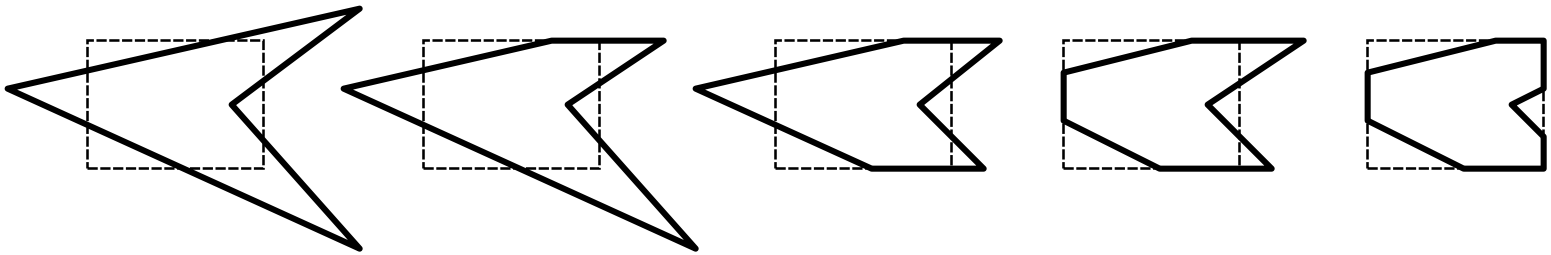
Polygon Clipping

- **Extended version of line clipping**
  - Condition: polygons have to remain closed
    - Filling, hatching, shading, ...

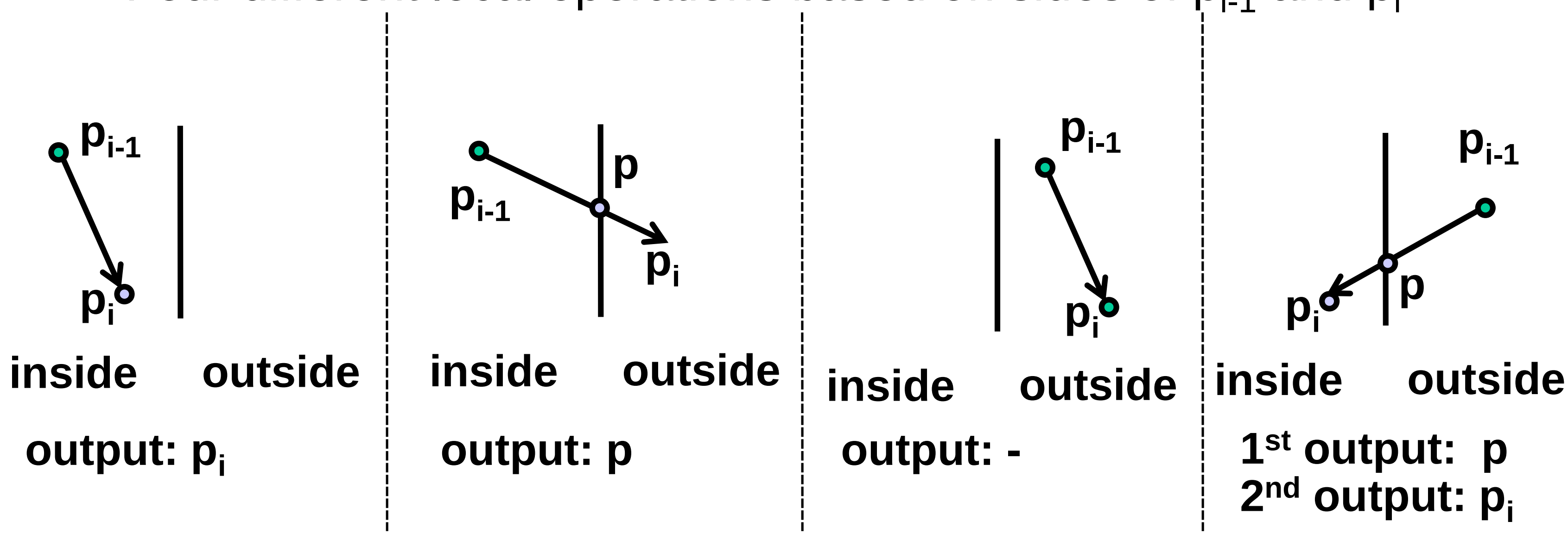


Sutherland-Hodgeman (1974)

- **Idea**
  - Iterative clipping against each edge in sequence



  - Four different local operations based on sides of p_{i-1} and p_i

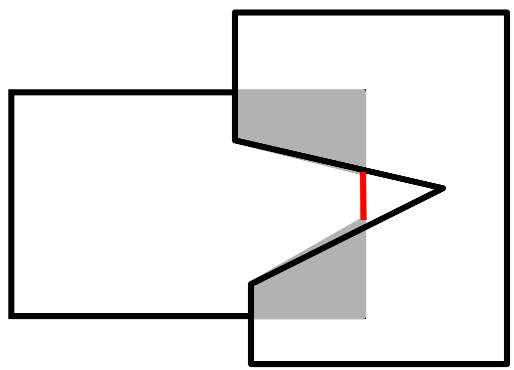


Enhancements

- **Recursive polygon clipping**
  - Pipelined Sutherland-Hodgeman

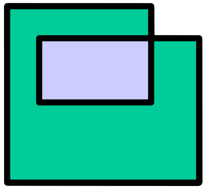
$p_0, p_1, \ldots \rightarrow$ Top $\rightarrow$ Bottom $\rightarrow$ Left $\rightarrow$ Right $\rightarrow p_0, p_1, \ldots$

- **Problems**
  - Degenerated polygons/edges
    - Elimination by post-processing, if necessary



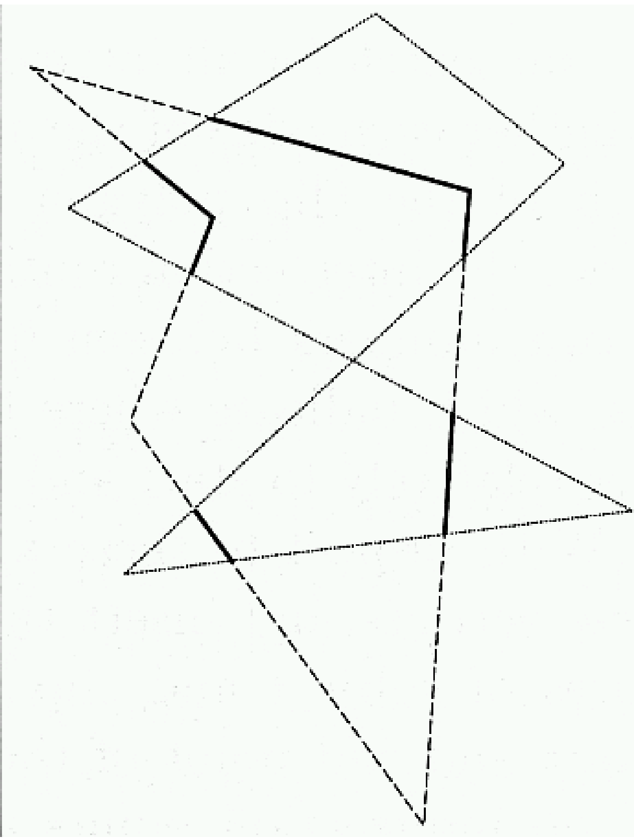
Other Clipping Algorithms

- **Weiler & Atherton (´77)**
  - Arbitrary concave polygons with holes against each other
- **Vatti (´92)**
  - Also with self-overlap
- **Greiner & Hormann (TOG ´98)**
  - Simpler and faster as Vatti
  - Also supports Boolean operations
  - Idea:
    - Odd winding number rule
      - Intersection with the polygon leads to a winding number ± 1
    - Walk along both polygons
    - Alternate winding number value
    - Mark point of entry and point of exit
    - Combine results

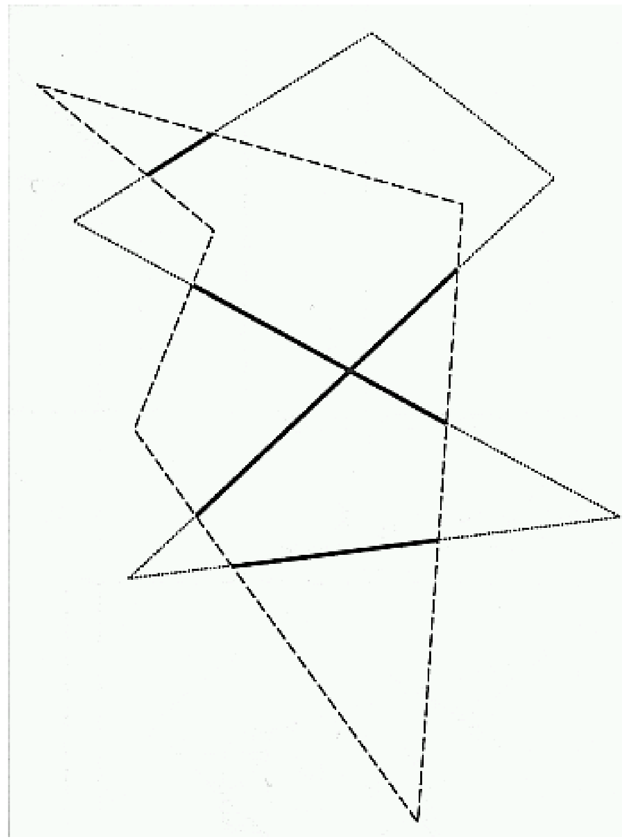


Non-zero WN: in
Even WN: out

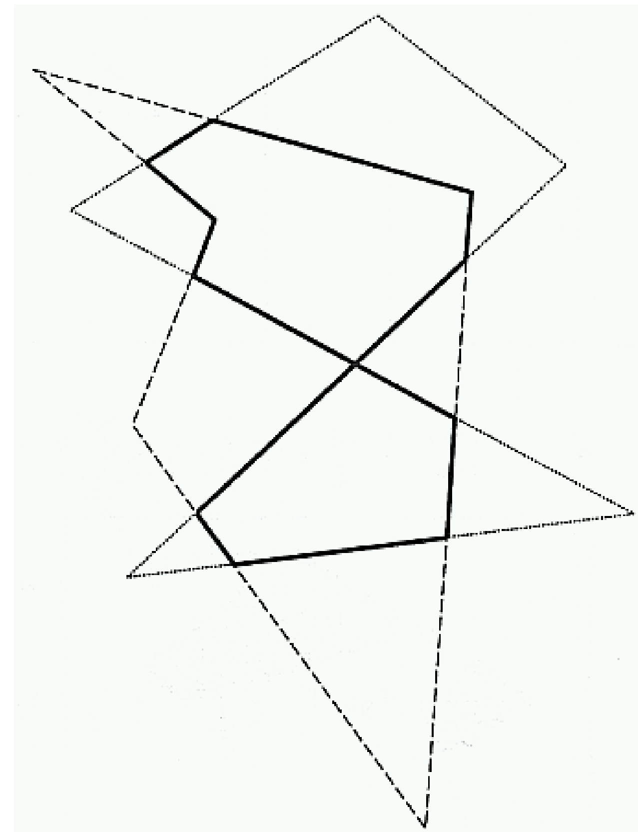
Greiner & Hormann



A in B



B in A



(A in B) ∪ (B in A)

3D Clipping agst. View Volume

- **Requirements**
  - Avoid unnecessary rasterization
  - Avoid overflow on transformation at fixed point!
- **Clipping against viewing frustum**
  - Enhanced Cohen-Sutherland with 6-bit outcode
  - After perspective division
    - $-1 < y < 1$
    - $-1 < x < 1$
    - $-1 < z < 0$
  - Clip against side planes of the canonical viewing frustum
  - Works analogously with Liang-Barsky or Sutherland-Hodgeman

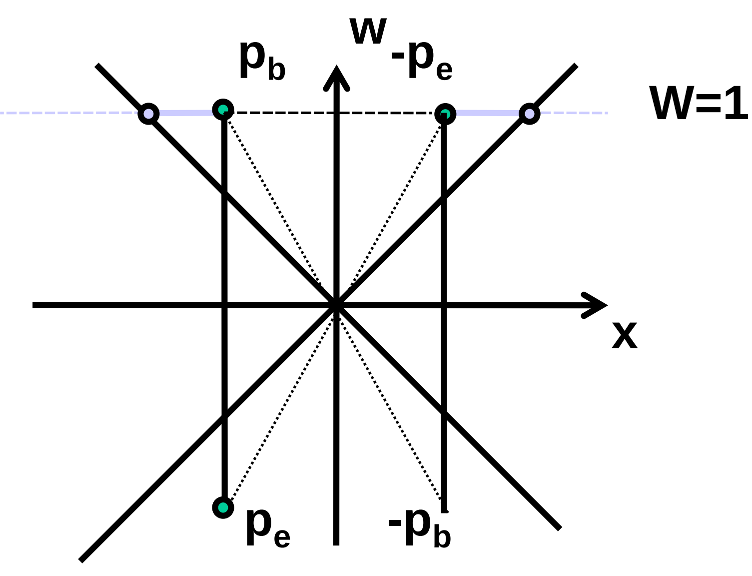
3D Clipping agst. View Volume

- **Clipping in homogeneous coordinates**
  - Use canonical view frustum, but avoid costly division by W
  - Inside test with a linear distance function (WEC)
    - Left: X / W > -1 ➔ W + X = $WEC_L(\underline{p})$ > 0
    - Top: Y / W < 1 ➔ W – Y = $WEC_T(\underline{p})$ > 0
    - Back: Z / W > -1 ➔ W + Z = $WEC_B(\underline{p})$ > 0
    - …
  - Intersection point calculation (before homogenizing)
    - Test: $WEC_L(\underline{p}_b) > 0$ and $WEC_L(\underline{p}_e) < 0$
    - Calculation:

$$WEC\big(p_b + t(p_e - p_b)\big) = 0$$
$$W_b + t(W_e - W_b) + X_b + t(X_e - X_b) = 0$$
$$t = \frac{W_b + X_b}{(W_b + X_b) - (W_e + X_e)} = \frac{WEC_L(p_b)}{WEC_L(p_b) - WEC_L(p_e)}$$

Problems with Homogen. Coord.

- **Negative w**
  - Points with w < 0 or lines with $w_b < 0$ and $w_e < 0$
    - Negate and continue
  - Lines with $w_b \cdot w_e < 0$ (NURBS)
    - Line moves through infinity
      - External „line“
    - Clipping two times
      - Original line
      - Negated line
    - Generates up to two segments



Practical Implementations

- **Combining clipping and scissoring**
  - Clipping is expensive and should be avoided
    - Intersection calculation
    - Variable number of new points, new triangles
  - Enlargement of clipping region
    - (Much) larger than viewport, but
    - Still avoiding overflow due to fixed-point representation
  - Result
    - Less clipping
    - Applications should avoid drawing objects that are outside of the viewport/viewing frustum
    - Objects that are partially outside will be implicitly clipped during rasterization
    - Slight penalty because they will still be processed (triangle setup)