Computer Graphics

- Light Transport -

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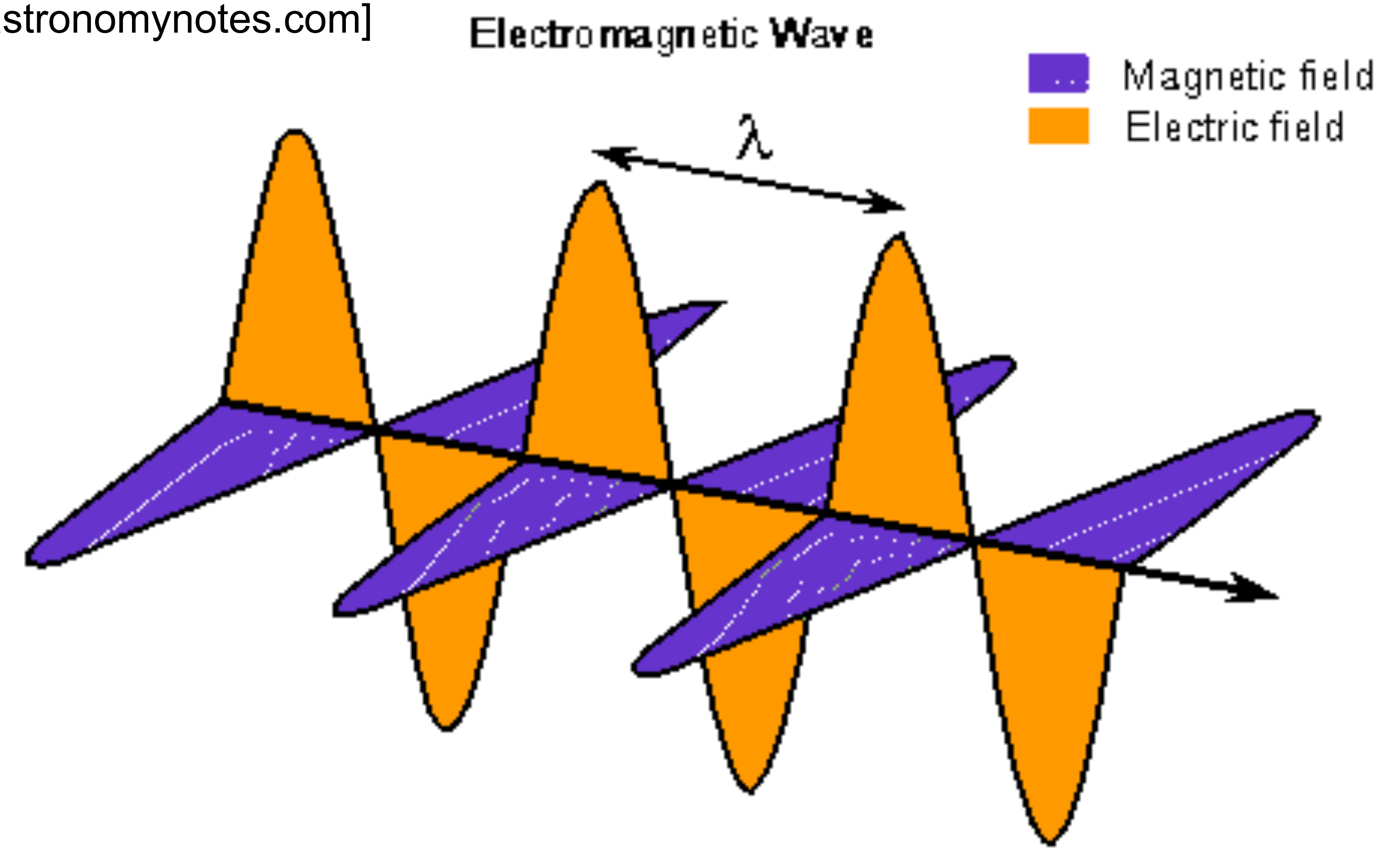
Overview

- **So far**
  - Nuts and bolts of ray tracing
- **Today**
  - Light
    - Physics behind ray tracing
    - Physical light quantities
    - Perception of light
    - Light sources
  - Light transport simulation
- **Next lecture**
  - Reflectance properties
  - Shading

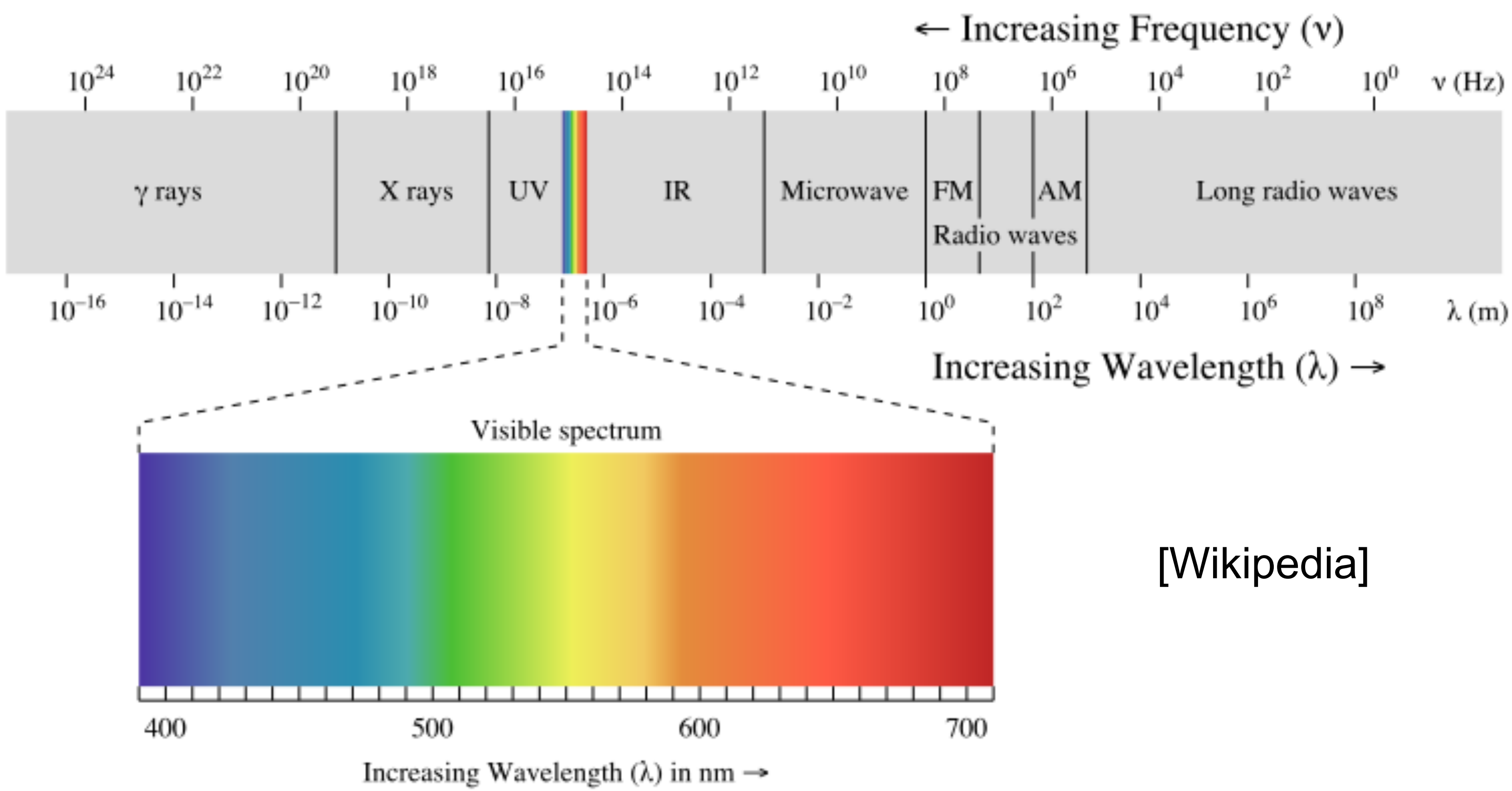
LIGHT

What is Light ?

[astronomynotes.com]



What is Light ?



[Wikipedia]

What is Light ?

- **Ray**
  - Linear propagation
  - Geometrical optics
- **Vector**
  - Polarization
  - **Jones Calculus**: matrix representation
- **Wave**
  - Diffraction, interference
  - **Maxwell equations**: propagation of light
- **Particle**
  - Light comes in discrete energy quanta: photons
  - **Quantum theory**: interaction of light with matter
- **Field**
  - Electromagnetic force: exchange of virtual photons
  - **Quantum Electrodynamics (QED)**: interaction between particles

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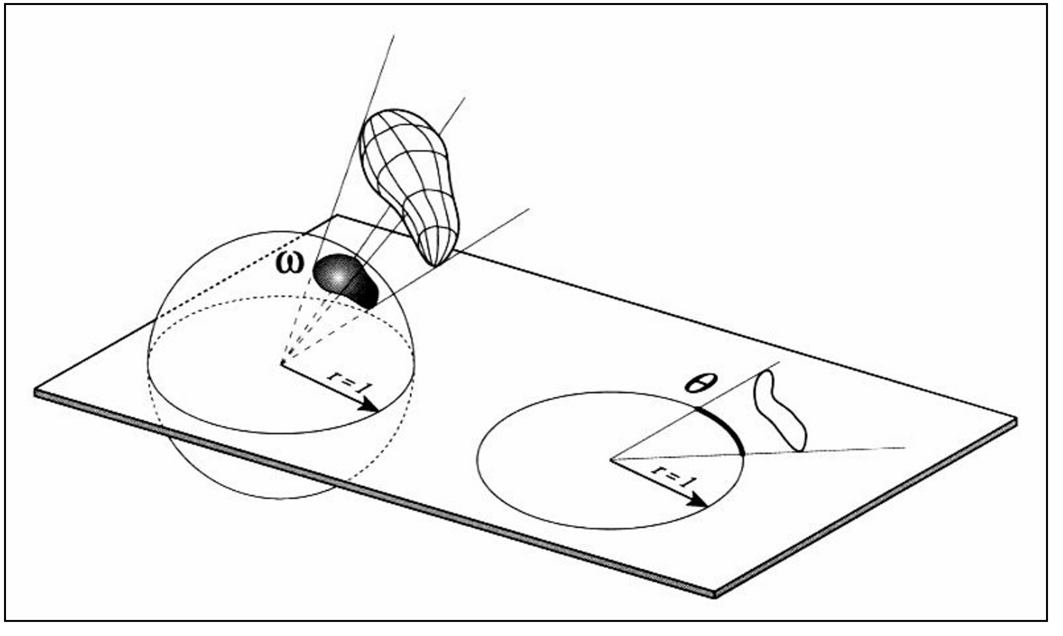
Light in Computer Graphics

- **Based on human visual perception**
  - Macroscopic geometry
  - Tristimulus color model
  - Psycho-physics: tone mapping, compression, …
- **Ray optics**
  - Macroscopic objects
  - Incoherent light
  - Light: scalar, real-valued quantity
  - Linear propagation
  - Superposition principle: light contributions add up linearly
  - No attenuation in free space
- **Limitations**
  - Microscopic structures ($\approx \lambda$): diffraction, interference
  - Polarization
  - Dispersion

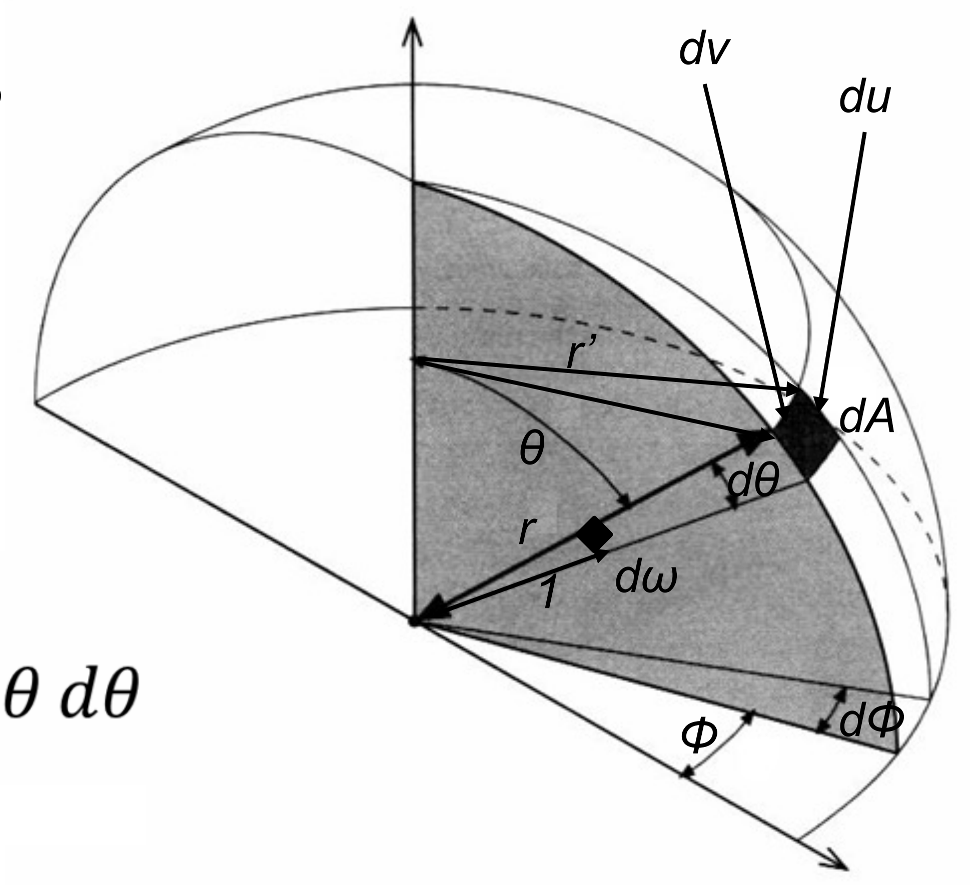
Angle and Solid Angle

- **The angle θ (in radians) subtended by a curve in the plane is the length of the corresponding arc on the unit circle: $l = \theta r = \theta$**
- **The solid angle Ω, $d\omega$ subtended by an object is the surface area of its projection onto the unit sphere**
  - Units for measuring solid angle: steradian [sr] (dimensionless)



Solid Angle in Spherical Coords

- **Infinitesimally small solid angle $d\omega$**
  - $du = r\, d\theta$
  - $dv = r'\, d\Phi = r \sin\theta\, d\Phi$
  - $dA = du\, dv = r^2 \sin\theta\, d\theta\, d\Phi$
  - $d\omega = dA / r^2 = \sin\theta\, d\theta\, d\Phi$



- **Finite solid angle**

$$\Omega = \int_{\phi_0}^{\phi_1} d\phi \int_{\theta_0(\phi)}^{\theta_1(\phi)} \sin\theta\, d\theta$$

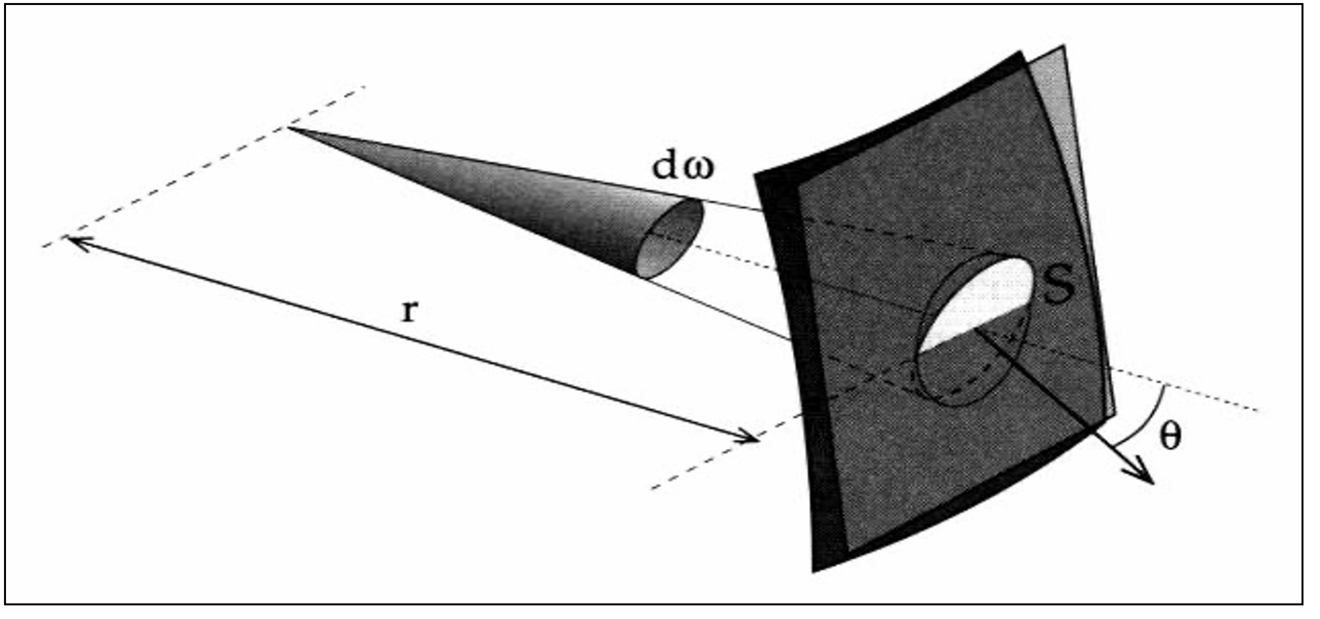
Solid Angle for a Surface

- The solid angle subtended by a small surface patch *S* with area *dA* is obtained (i) by projecting it orthogonal to the vector *r* to the origin:

$$dA\,cos\,\theta$$

and (ii) dividing by the distance to the origin squared: $\mathrm{d}\omega = \frac{\mathrm{d}A \cos\theta}{r^2}$

$$\Omega = \iint_S \frac{\vec{r}\cdot\vec{n}}{r^3}\,dA$$



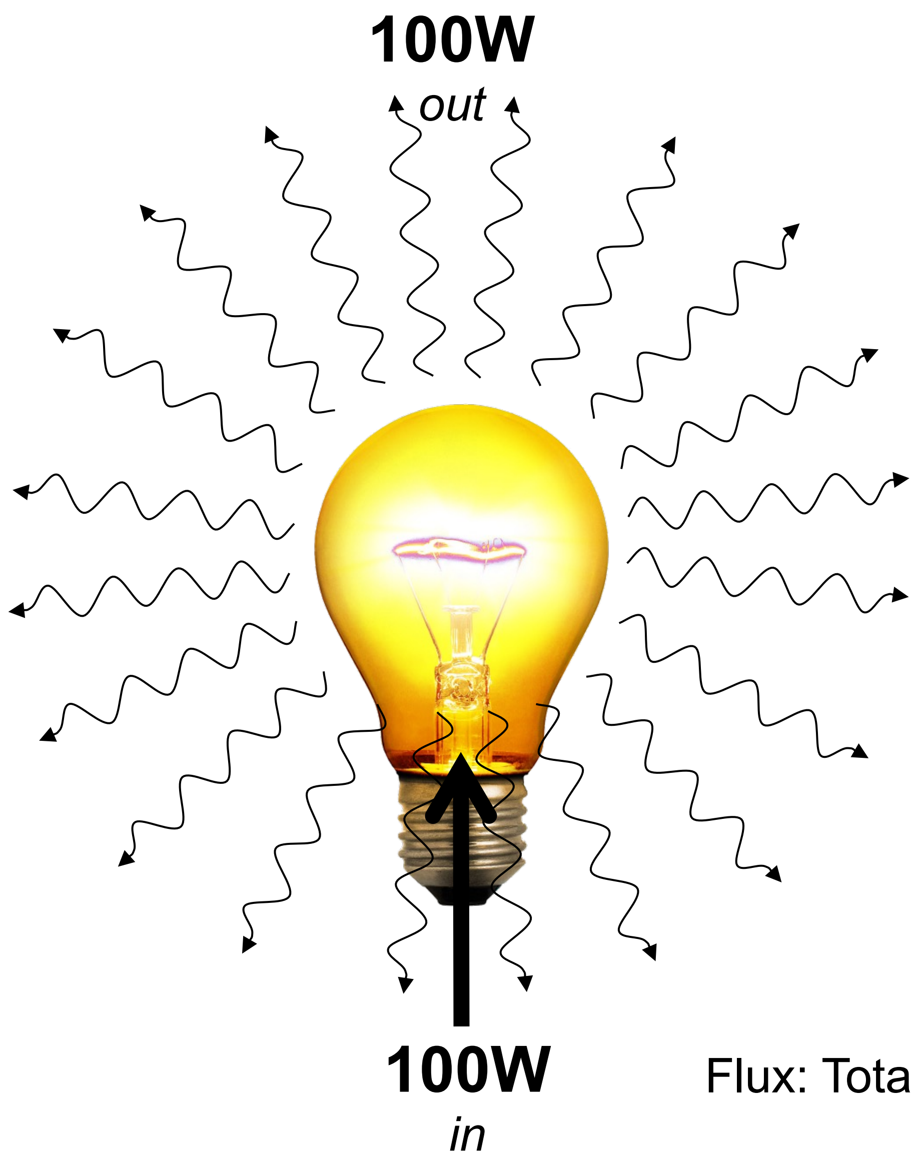
Radiometry

- **Definition:**
  - Radiometry is the science of measuring radiant energy transfers. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral photometers.

- **Radiometric Quantities**

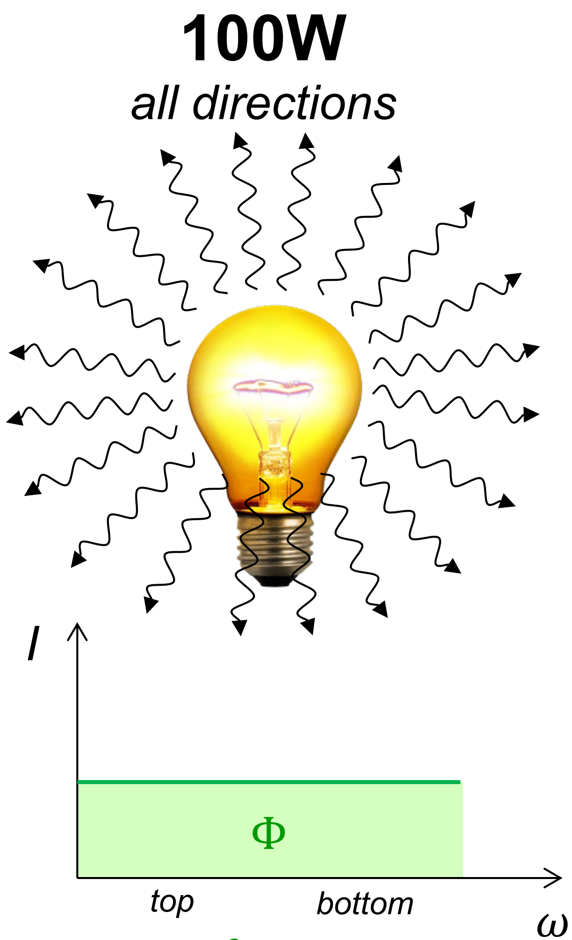
– Energy	[J]	Q	(Photons Energy = $n \cdot h\nu$)
– Radiant power	[watt = J/s]	Φ	(Total Flux)
– Intensity	[watt/sr]	I	
– Irradiance	[watt/m^2]	E	
– Radiosity	[watt/m^2]	B	
– Radiance	[watt/(m^2 sr)]	L	

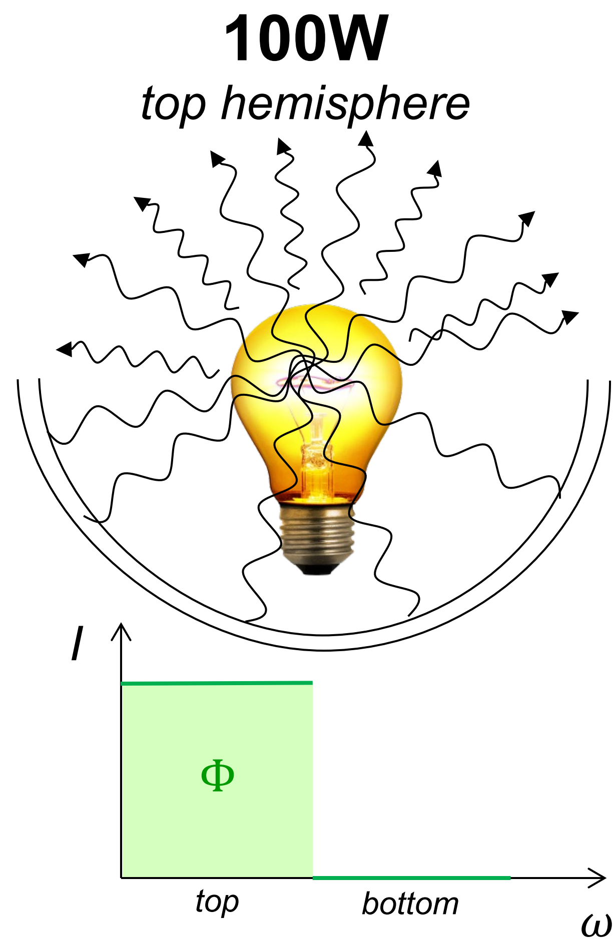
Radiant flux



Flux: Total energy in a region per unit time

Intensity



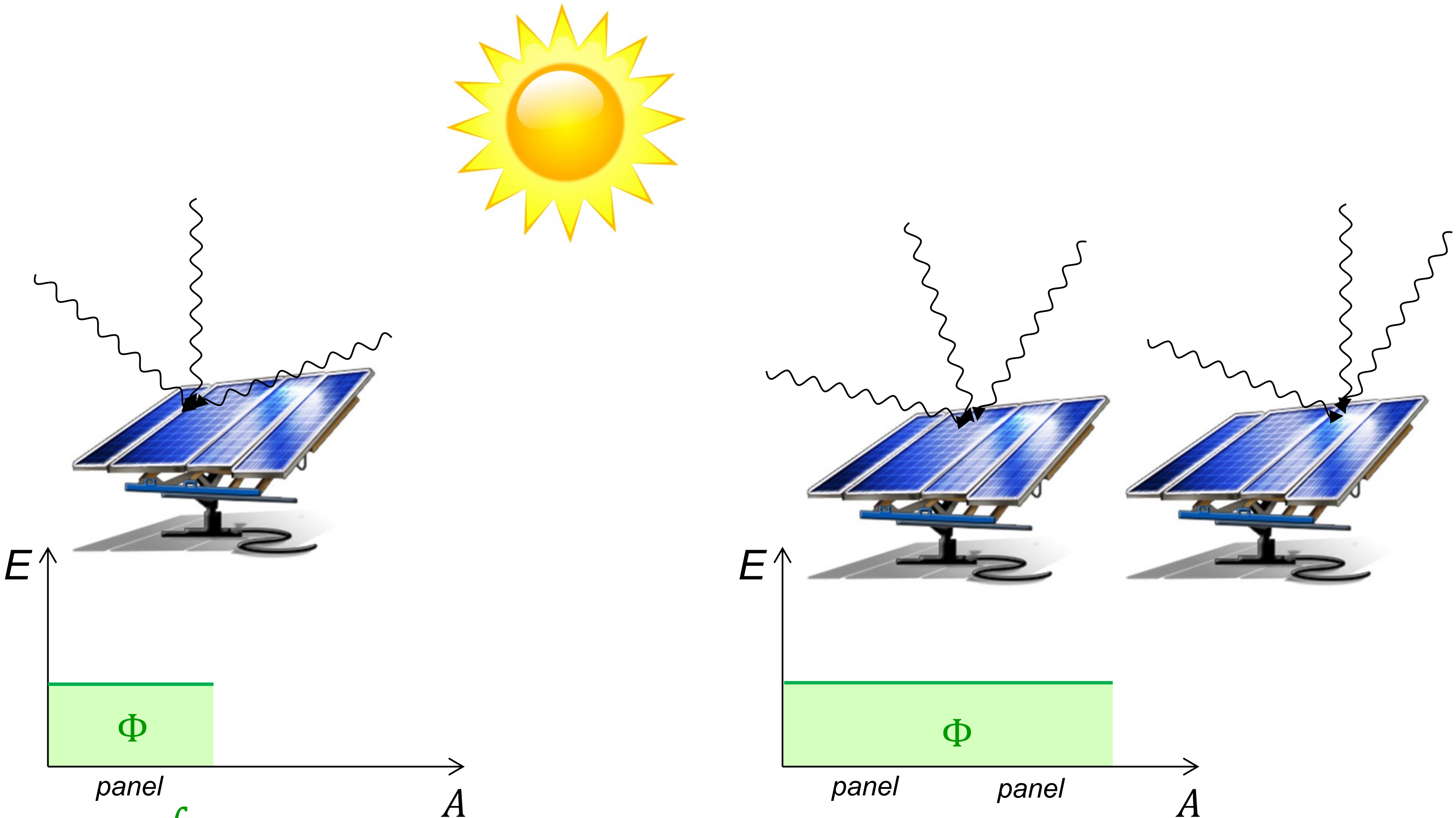


$$\Phi = \int_{\Omega} I \cdot d\omega$$

$$I = \frac{\mathrm{d}\Phi}{d\omega}$$

Intensity: Flux density per solid angle

Irradiance

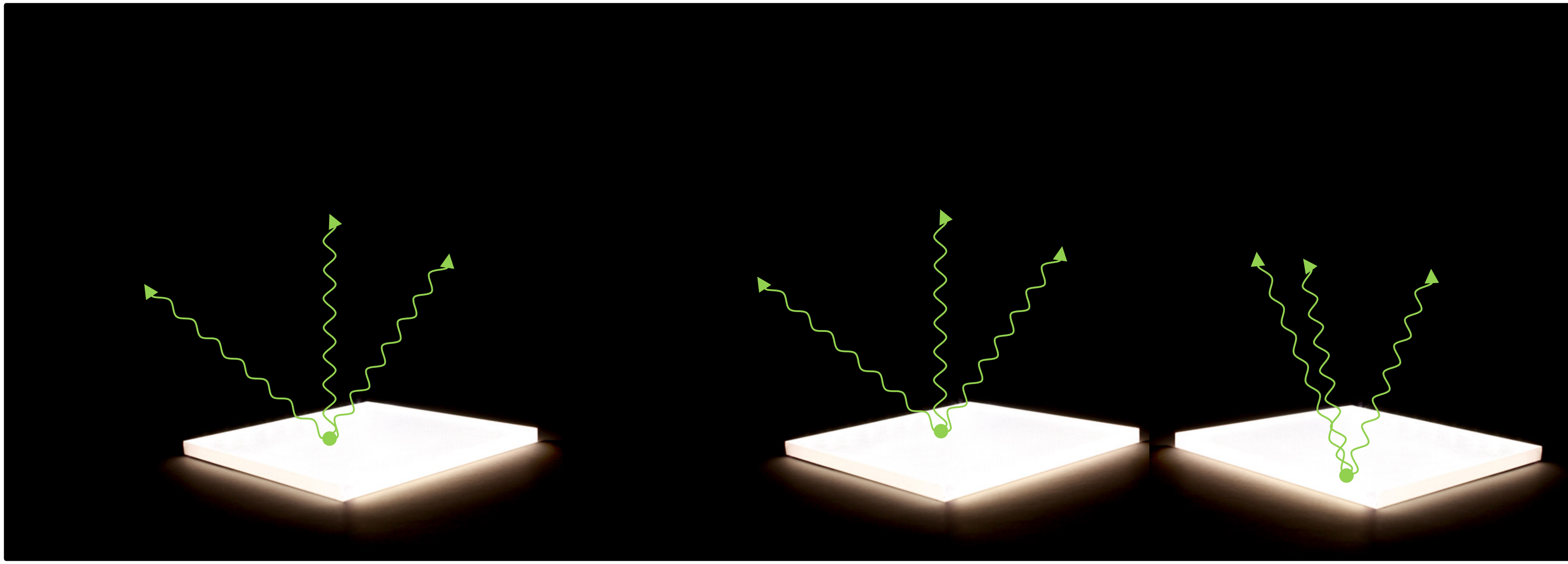


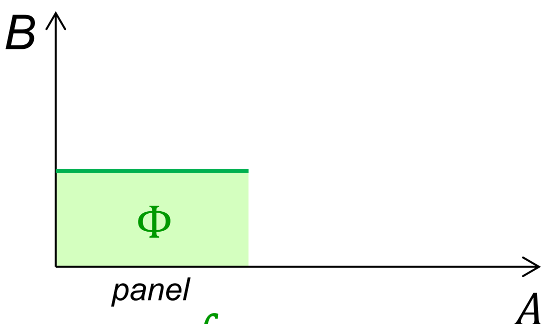
$$\Phi = \int_A E \cdot dA$$

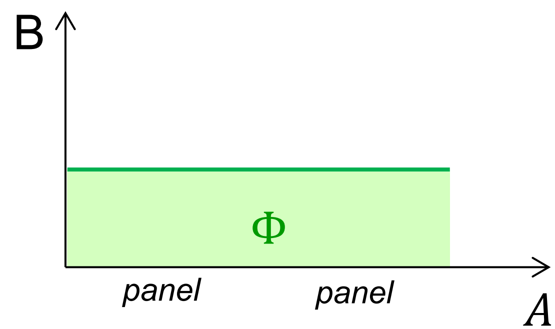
$$E = \frac{d\Phi}{dA}$$

Irradiance: Incoming flux density per surface

Radiosity







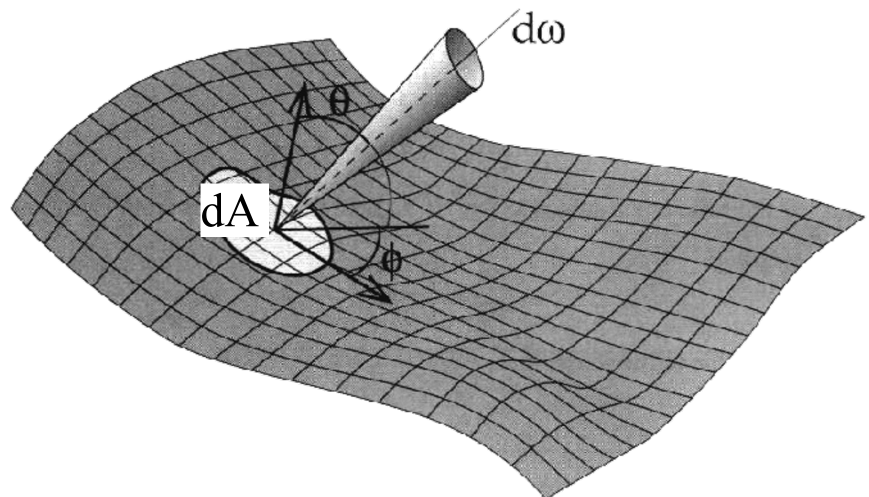
$$\Phi = \int_A B \cdot dA$$

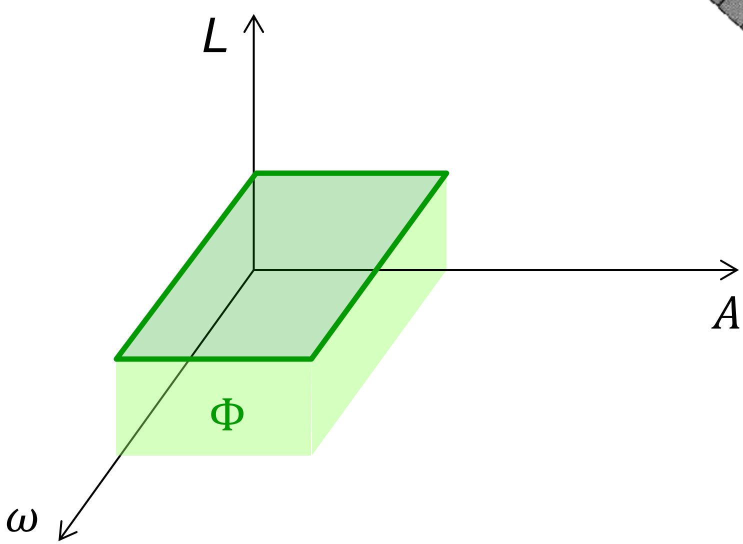
$$B = \frac{d\Phi}{dA}$$

Radiosity: Outgoing flux density per surface

Radiance

ω



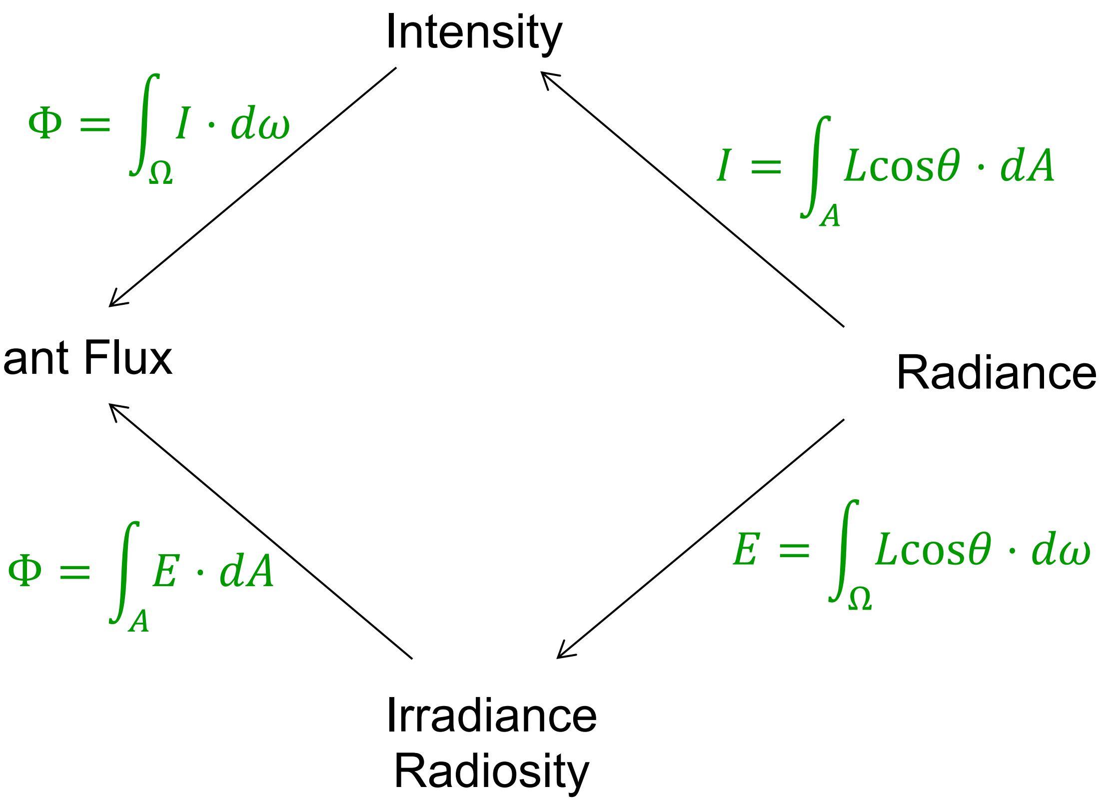


$$\Phi = \int_\Omega \int_A L \cdot d\omega dA_\perp = \int_\Omega \int_A L\cos\theta \cdot d\omega dA_\perp$$

$$L = \frac{\mathrm{d}^2\Phi}{d\omega dA_\perp} = \frac{\mathrm{d}^2\Phi}{d\omega dA\cos\theta}$$

Radiosity: Outgoing flux density
per surface
per solid angle

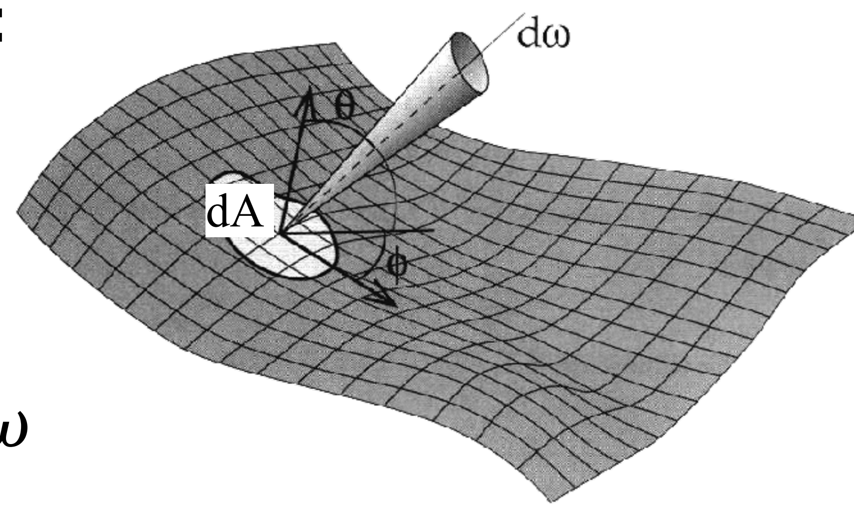
Radiance



Intensity
$\Phi = \int_{\Omega} I \cdot d\omega$
$I = \int_{A} L\cos\theta \cdot dA$
Radiant Flux
Radiance
$\Phi = \int_{A} E \cdot dA$
$E = \int_{\Omega} L\cos\theta \cdot d\omega$
Irradiance
Radiosity

Radiometric Quantities: Radiance

- **Radiance is used to describe radiant energy transfer**
- **Radiance *L* is defined as**
  - The power (flux) traveling at some point *x*
  - In a specified direction $\omega = (\theta, \varphi)$
  - Per unit area perpendicular to the direction of travel
  - Per unit solid angle
- **Thus, the differential power $d^2\Phi$ radiated through the differential solid angle $d\omega$, from the projected differential area $dA \cos\theta$ is:** ω



$$d^2\Phi = L(x, \omega) dA \cos\theta \, d\omega$$

Radiometric Quantities: Irradiance

- Irradiance E is defined as the **total power per unit area** (flux density) incident onto a surface. To obtain the total flux incident to *dA*, the incoming radiance L_i is integrated over the upper hemisphere Ω_+ above the surface:

$$E \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega_+} L_i(x,\omega)\cos\theta\, d\omega\right] dA$$

$$E = \int_{\Omega_+} L_i(x,\omega)\cos\theta\, d\omega = \int_0^{\frac{\pi}{2}}\int_0^{2\pi} L_i(x,\omega)\cos\theta\sin\theta\, d\theta d\phi$$

Radiometric Quantities: Radiosity

- **Radiosity B** is defined as the **total power per unit area** (flux density) **exitant from** a surface. To obtain the total flux incident to *dA*, the outgoing radiance L_o is integrated over the upper hemisphere Ω_+ above the surface:

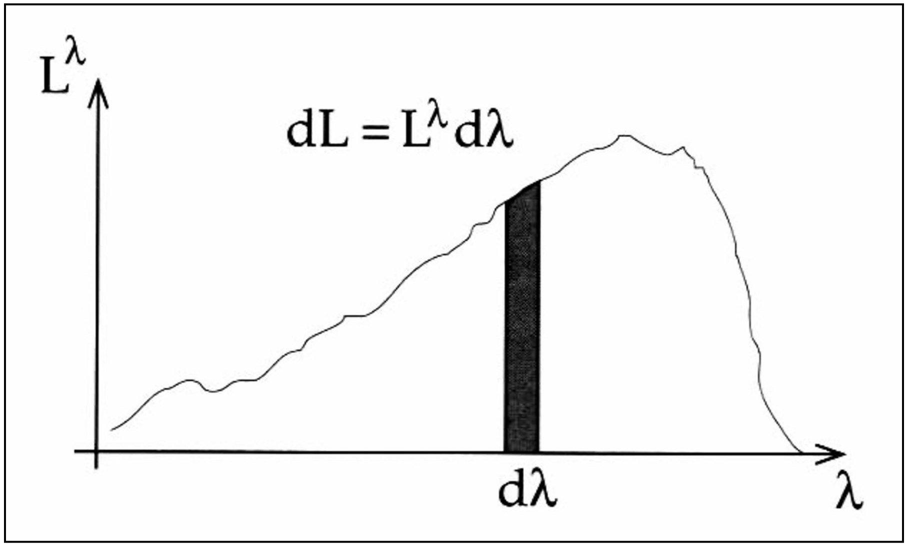
$$B \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega_+} L_o(x,\omega)\cos\theta\, d\omega\right] dA$$

$$B = \int_{\Omega_+} L_o(x,\omega)\cos\theta\, d\omega = \int_0^{\frac{\pi}{2}}\int_0^{2\pi} L_o(x,\omega)\cos\theta\sin\theta\, d\theta d\phi$$

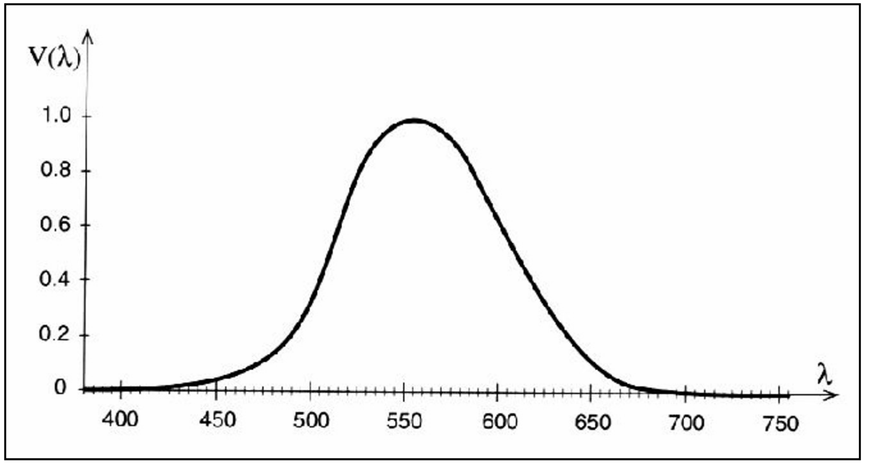
Spectral Properties

- **Wavelength**
  - Light is composed of electromagnetic waves
  - These waves have different frequencies and wavelengths
  - Most transfer quantities are continuous functions of wavelength
- **In graphics**
  - Each measurement *L(x,ω)* is for a discrete band of wavelength only
    - Often some abstract R, G, B (but see later)



Photometry

- The human eye is sensitive to a limited range of wavelengths
  - Roughly from 380 nm to 780 nm
- Our visual system responds differently to different wavelengths
  - Can be characterized by the *Luminous Efficiency Function* V(λ)
  - Represents the average human spectral response
  - Separate curves exist for light and dark adaptation of the eye
- Photometric quantities are derived from radiometric quantities by *integrating* them against this function



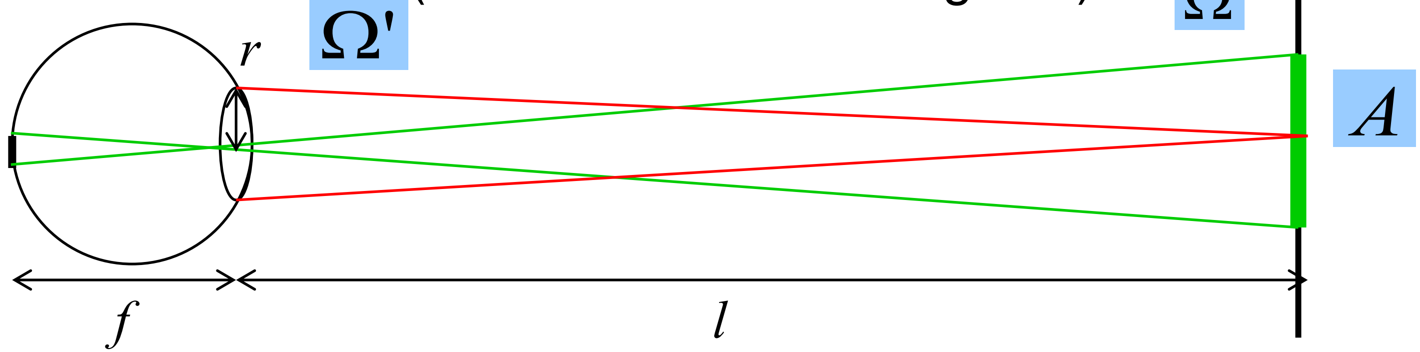
Radiometry vs. Photometry

Physics-based quantities **Perception-based quantities**

	Radiometry	→	Photometry	
W	Radiant power	→	Luminous power	Lumens (lm)
W/m^2	Radiosity Irradiance	→	Luminosity Illuminance	Lux (lm/m^2)
$W/m^2/sr$	Radiance	→	Luminance	cd/m^2 ($lm/m^2/sr$)

Perception of Light

(1 arcminute = 1/60 degrees)



photons / second = **flux = energy / time = power** Φ

rod sensitive to flux

angular extent of rod = **resolution** (≈ 1 arcminute²) Ω

projected rod size = **area** $A \approx l^2 \cdot \Omega$

angular extent of pupil aperture (r ≤ 4 mm) = **solid angle** $\Omega' \approx \pi \cdot r^2 / l^2$

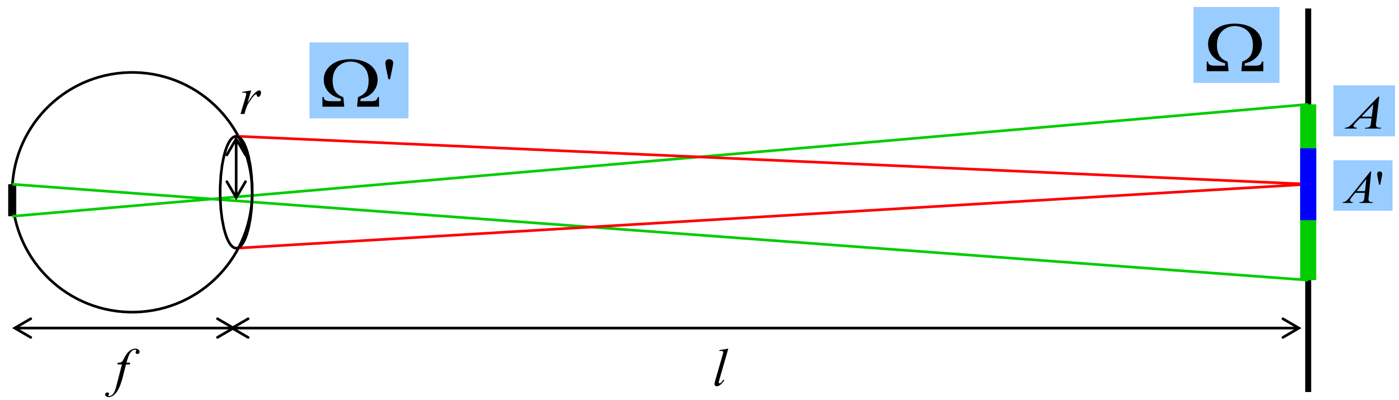
flux proportional to area and solid angle $\Phi = L\, A\, \Omega'$

radiance = flux per unit area per unit solid angle $L = \frac{\Phi}{\Omega' \cdot A}$

The eye detects radiance

As *l* increases: $\Phi_0 = L \cdot l^2 \cdot \Omega \cdot \pi \frac{r^2}{l^2} = L \cdot \mathrm{const}$

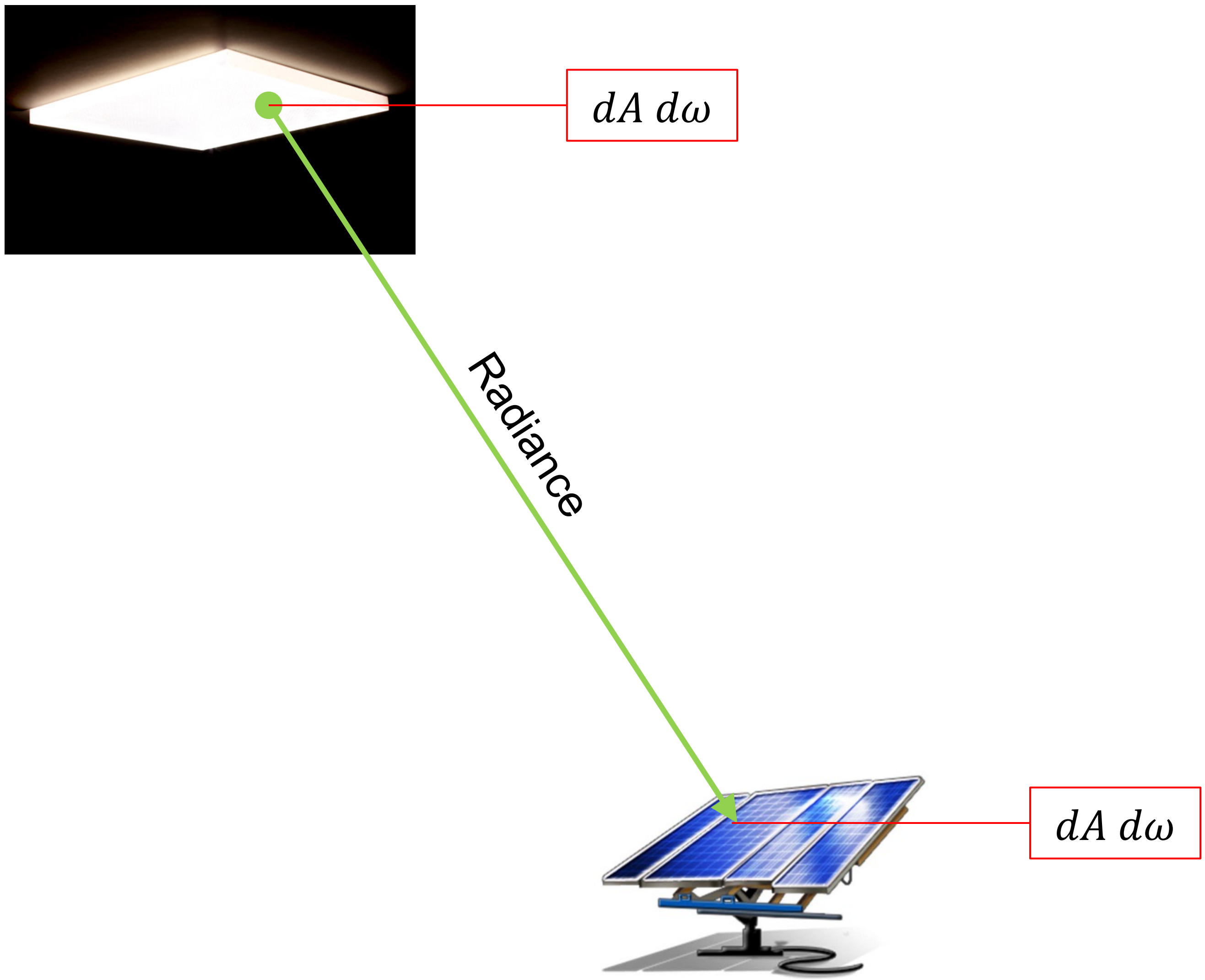
Brightness Perception



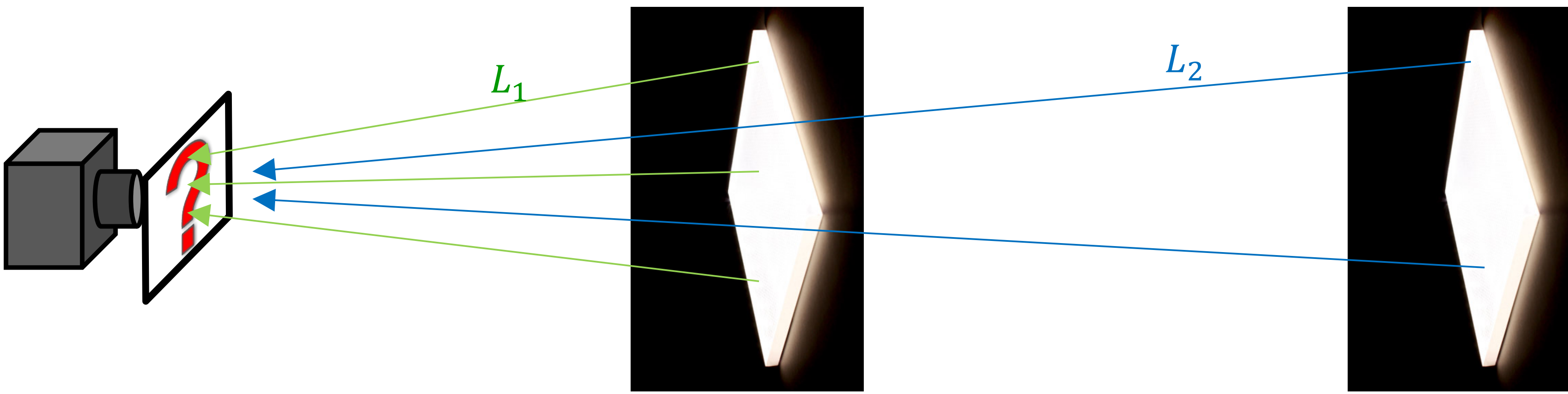
- $A' > A$: photon flux per rod stays constant
- $A' < A$: photon flux per rod decreases

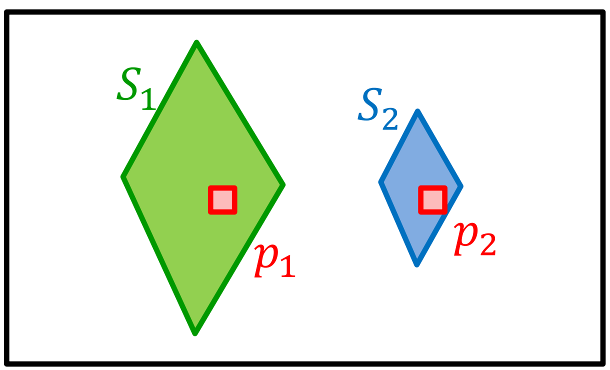
Where does the Sun turn into a star ?
– Depends on apparent Sun disc size on retina
– Photon flux per rod stays the same on Mercury, Earth or Neptune
– Photon flux per rod decreases when $\Omega' < 1$ arcminute^2 (beyond Neptune)

Radiometry in ray tracing



Radiometry in ray tracing





$$\frac{S_1}{S_2} \approx \left(\frac{d_2}{d_1}\right)^2$$

Each pixel:

$$L_1 = L_2$$

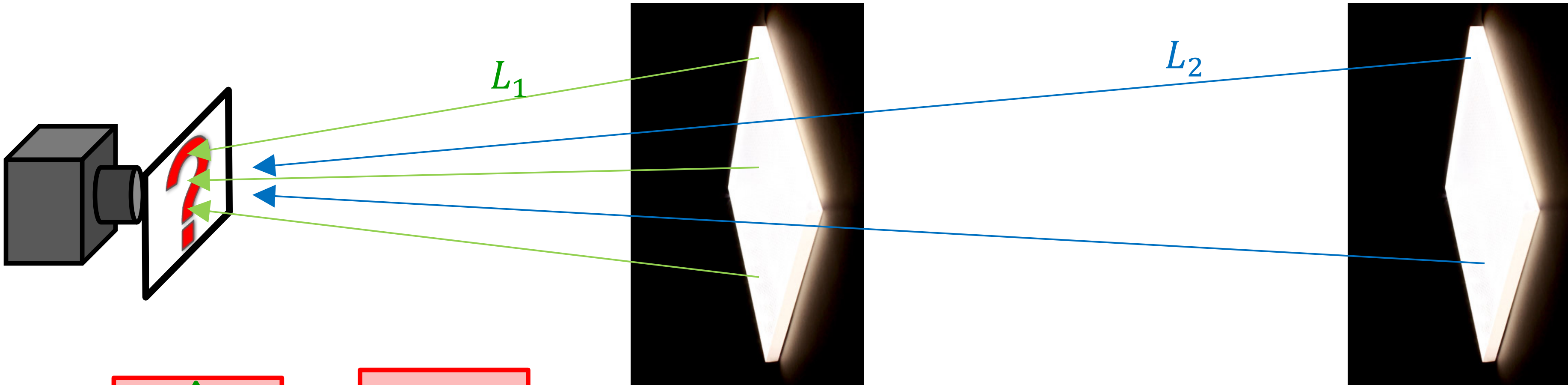
$$\text{color}[p_1] = \text{color}[p_2]$$

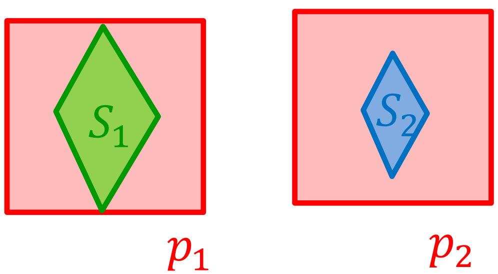
Total area:

$$\Phi_1 = \int_\Omega \int_{A_1} L \cdot d\omega dA_\perp \approx \int_{A_1} E \cdot dA = ES_1$$

$$\Phi_2 = \int_\Omega \int_{A_2} L \cdot d\omega dA_\perp \approx \int_{A_2} E \cdot dA = ES_2$$

Radiometry in ray tracing





Each pixel:

$$L_1 = L_2$$

$$\text{color}[p_1] = L_1 \frac{S_1}{S_{p_1}}$$

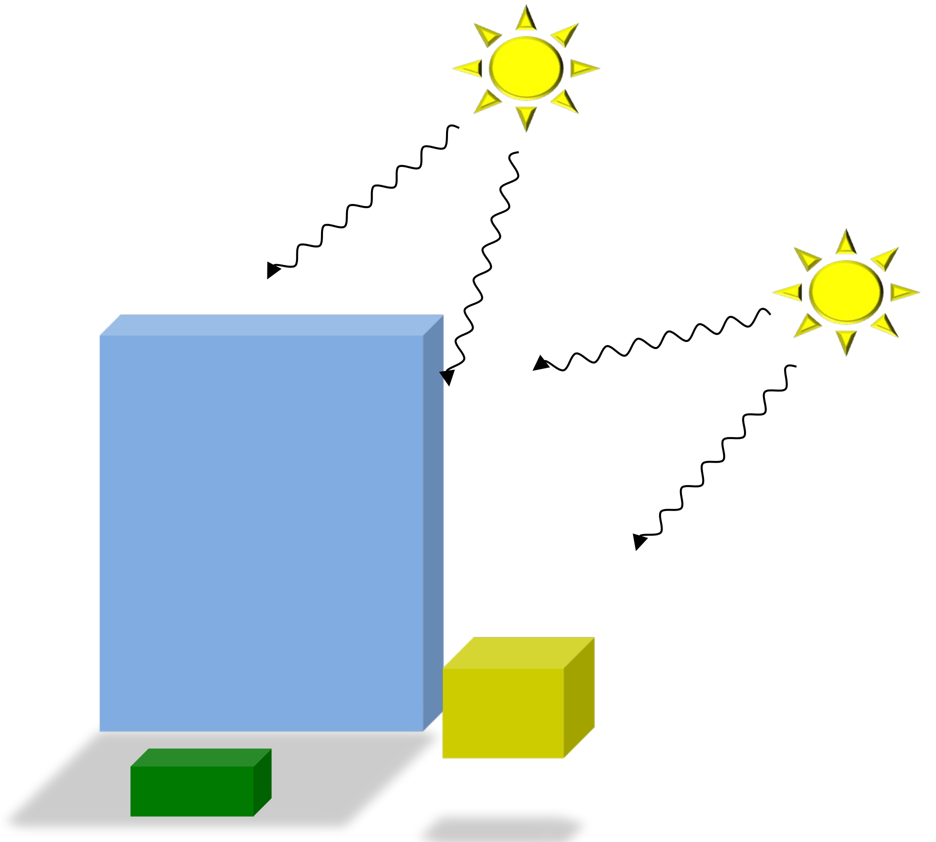
$$\text{color}[p_2] = L_2 \frac{S_2}{S_{p_2}}$$

$$\frac{S_1}{S_2} \sim \left(\frac{d_2}{d_1}\right)^2$$

LIGHT TRANSPORT

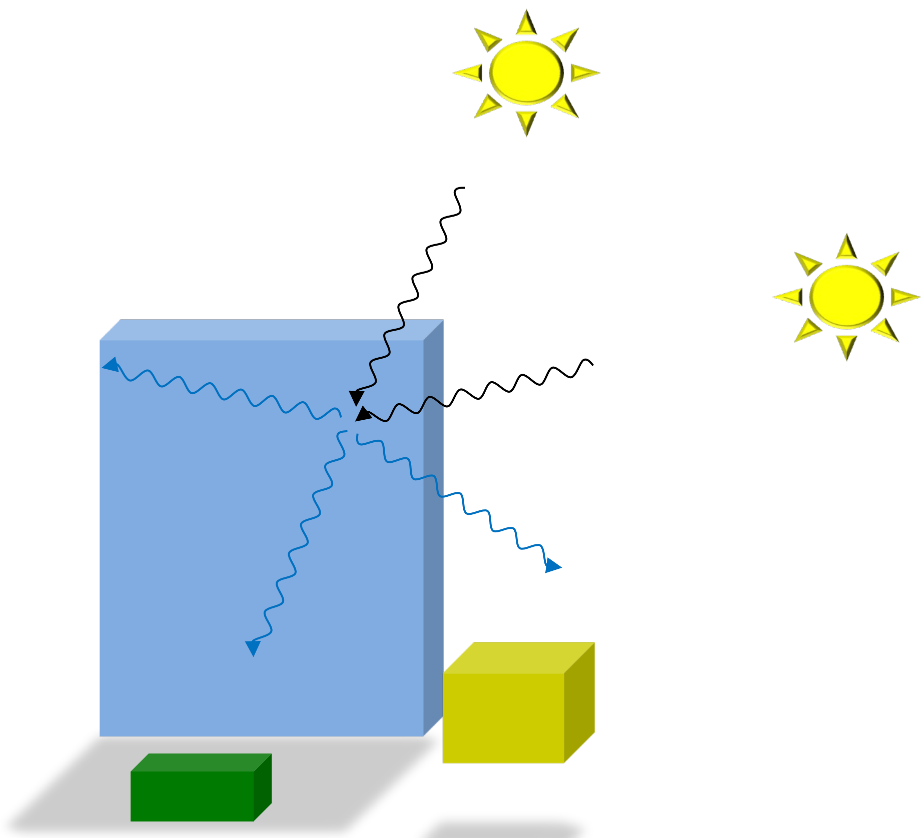
Light Transport in a Scene

- **Scene**
  - Lights (emitters)
  - Object surfaces (partially absorbing)



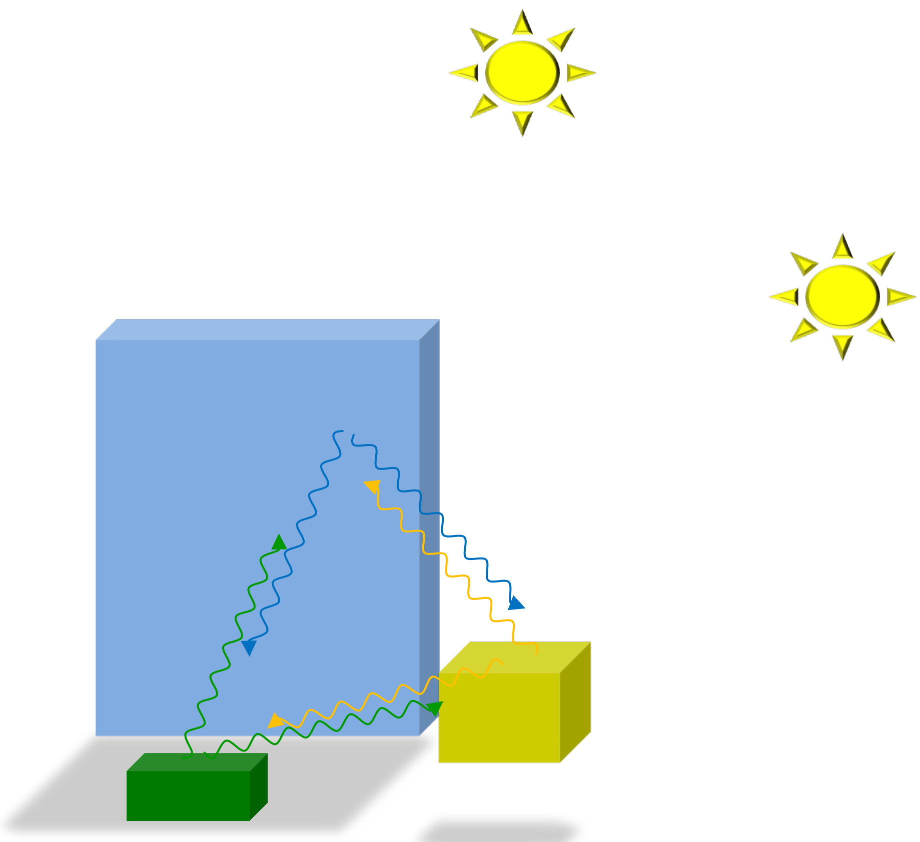
Light Transport in a Scene

- **Illuminated object surfaces become emitters, too!**
  - Radiosity = Irradiance – absorbed photons flux density
    - Radiosity: photons per second per m^2 leaving surface
    - Irradiance: photons per second per m^2 incident on surface



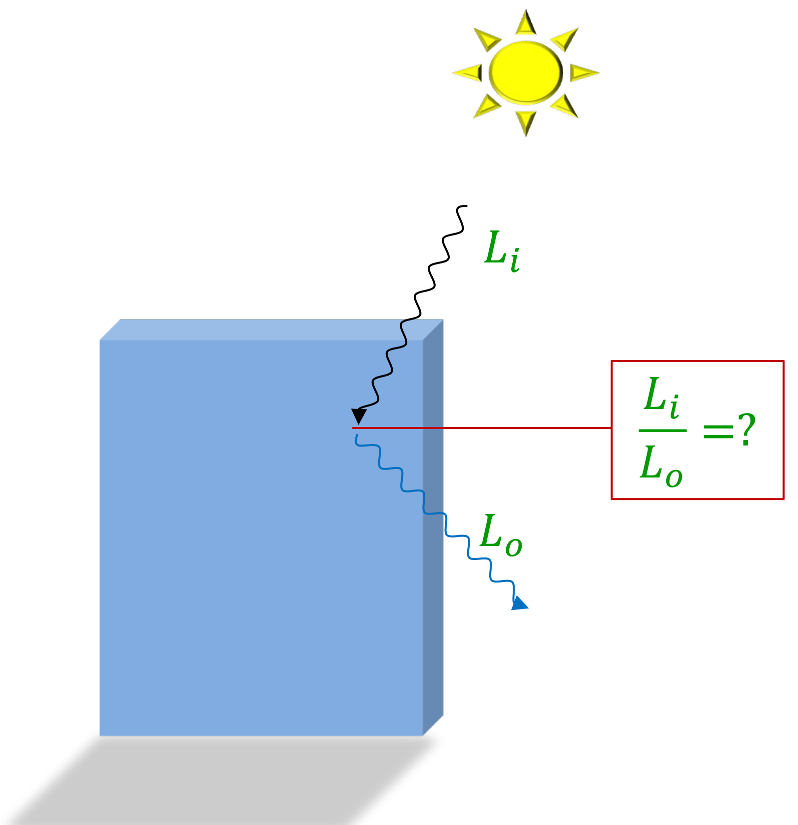
Light Transport in a Scene

- **Light bounces between all mutually visible surfaces**
- **Dynamic energy equilibrium**
  - Emitted photons = absorbed photons (+ escaping photons)
    → Global Illumination



Light Transport in a Scene

- **Light interaction with surfaces**
  - Incident angle
  - Material
    - → BRDF: bidirectional reflectance distribution function



Light Transport in a Scene

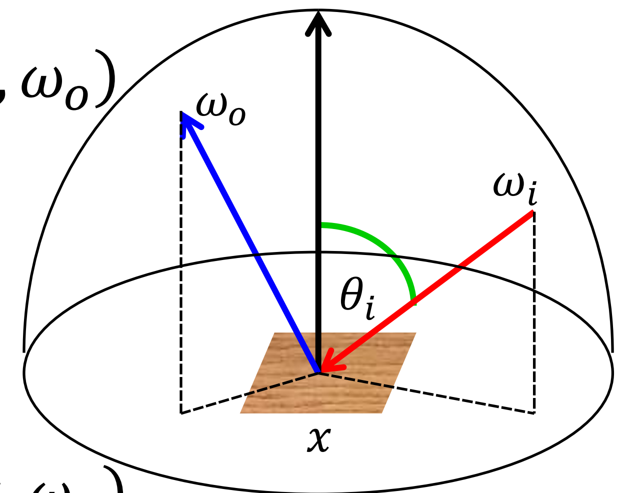
- **Outgoing radiance proportional to:**
  - Incoming radiance $L_o \sim L_i$
  - Incident angle $L_o \sim \cos\theta$
  - Material reflectance (BRDF) $L_o \sim f_r$
  - Material self emission $L_o \sim L_e$
- **Rendering equation:**

$$L_o = L_e + \int_{\Omega_+} f_r L_i \cos\theta_i \, d\omega_i$$

(Surface) Rendering Equation

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos\theta_i \, d\omega_i$$

- **Visible surface radiance** $L_o(x, \omega_o)$
  - Surface position x
  - Outgoing direction ω_o
- **Incoming illumination direction** ω_i



- **Self-emission** $L_e(x, \omega_o)$
- **Reflected light**
  - Incoming radiance from all directions $L_i(x, \omega_i)$
  - Direction-dependent reflectance (BRDF: bidirectional reflectance distribution function) $f_r(\omega_i, x, \omega_o)$

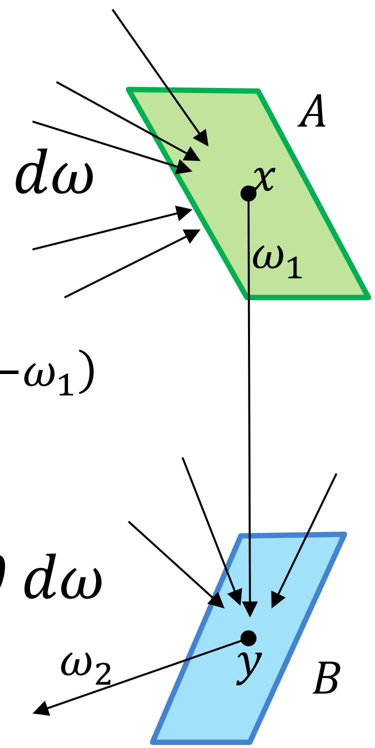
(Surface) Rendering Equation

- **Fredholm integral equation of 2nd kind**
  - Unknown radiance appears both on the left-hand side and inside the integral
  - Numerical methods necessary to compute approximate solution

$$L_A(x,\omega_1) = L_{A_e}(x,\omega_1) + \int_{\Omega_+} f_A(\omega,x,\omega_1)L(x,\omega)\cos\theta\, d\omega$$

$$L_A(x,\omega_1) = L(\mathrm{RT}(x,\omega_1),-\omega_1)$$

$$L_B(y,\omega_2) = L_{B_e}(y,\omega_2) + \int_{\Omega_+} f_B(\omega,y,\omega_2)L(y,\omega)\cos\theta\, d\omega$$



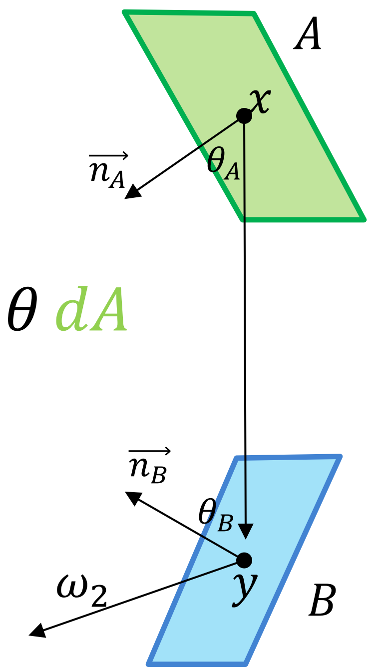
(Surface) Rendering Equation

- **Reparameterization over surfaces**
  - Represent receiver's $d\omega$ as emiter's dA.

$$L_B(y, \omega_2) = L_{B_e}(y, \omega_2) + \int_{\Omega_+} f_B(\omega, y, \omega_2) L(x, \omega) \cos\theta \, d\omega$$

$$\downarrow$$

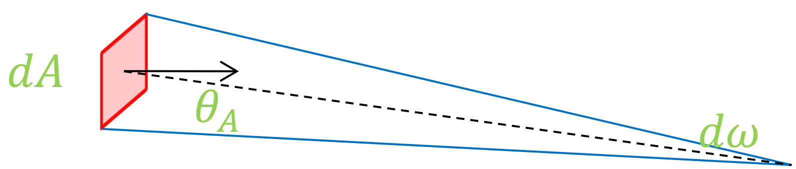
$$L_B(y, \omega_2) = L_{B_e}(y, \omega_2) + \int_A f_B(\omega_{yx}, y, \omega_2) \; ? \; \cos\theta \, dA$$



(Surface) Rendering Equation

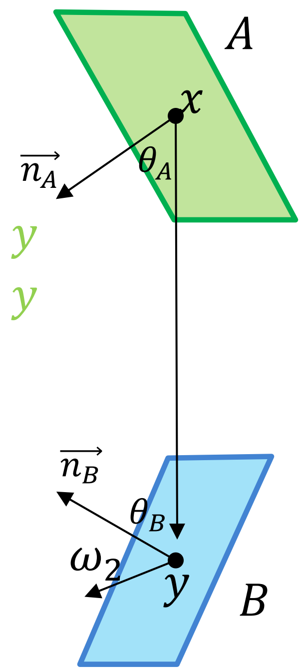
- **Reparameterization over surfaces**
  - Represent receiver's $d\omega$ as emiter's dA
  - Check visilbity

$$L_B(y,\omega_2) = L_{B_e}(y,\omega_2) + \int_{\Omega_+} f_B(\omega, y, \omega_2) L(x,\omega) \cos\theta \, d\omega$$



$$d\omega = \frac{\cos\theta_A}{\|x-y\|^2} dA$$

$$V(x,y) = \begin{cases} 0 & \Leftrightarrow \; RT(x,\omega_1) \neq y \\ 1 & \Leftrightarrow \; RT(x,\omega_1) = y \end{cases}$$



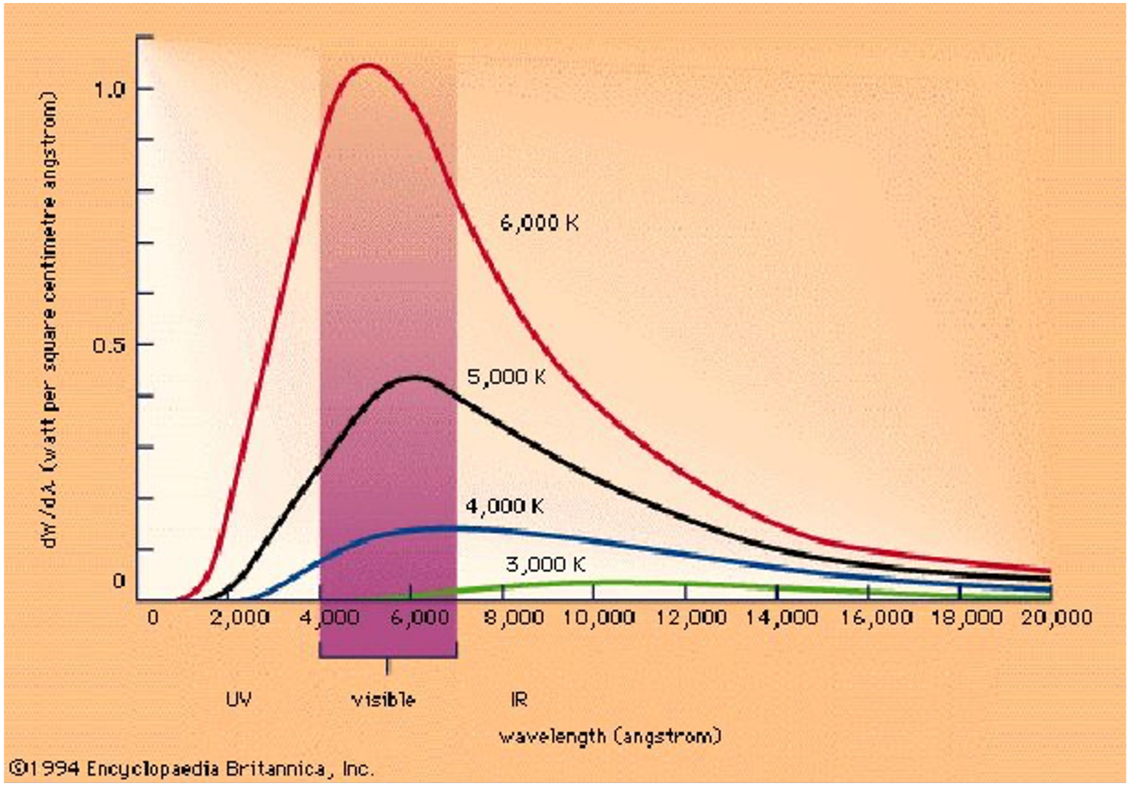
$$L_B(y,\omega_2) = L_{B_e}(y,\omega_2) +$$

$$\int_A f_B(\omega_{yx}, y, \omega_2) L(y, \omega_{yx}) V(x,y) \frac{\cos\theta_B \cos\theta_A}{\|x-y\|^2} dA$$

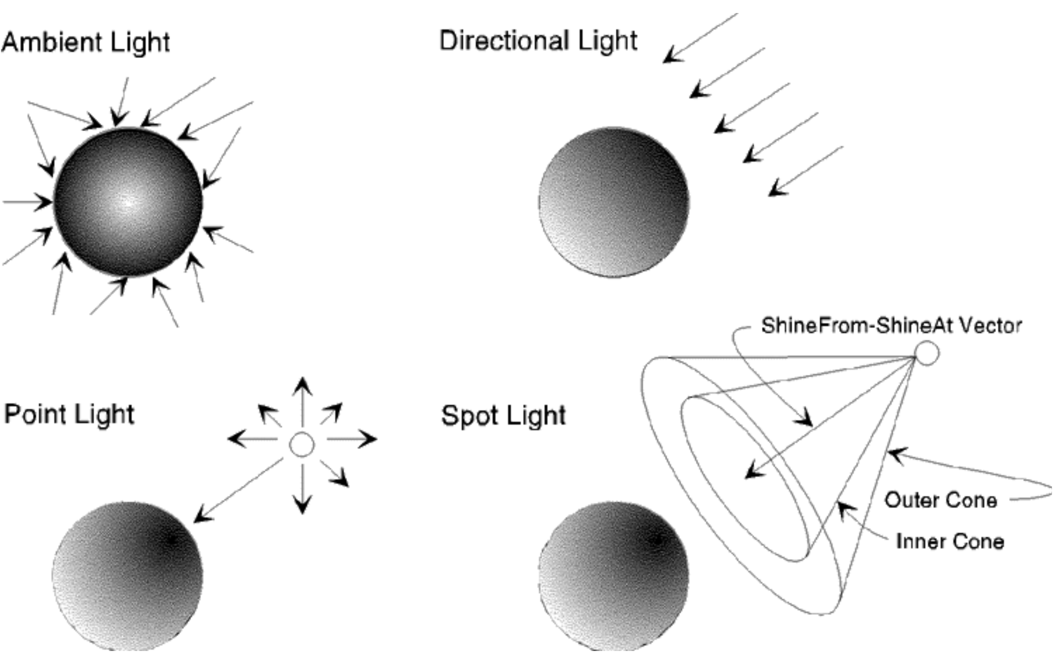
LIGHT SOURCES

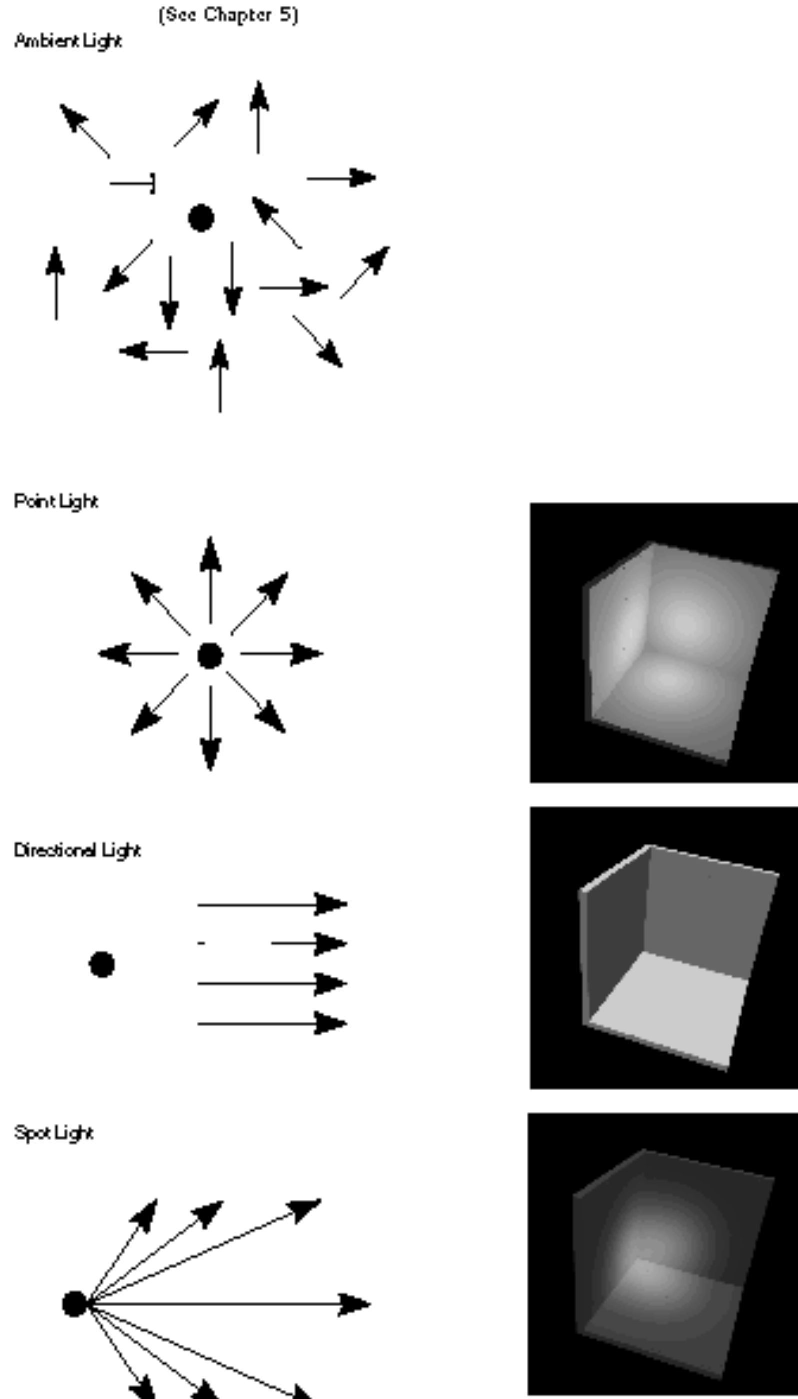
Light Specifications

- **Emitted Power Φ_e**
  - Total brightness
- **Spectral Distribution**
  - Continuous thermal spectrum
  - Discrete spectral lines
- **Approximation**
  - RGB color



Light Types


Ambient Light
Directional Light
Point Light
Spot Light
ShineFrom-ShineAt Vector
Outer Cone
Inner Cone


(See Chapter 5)
Ambient Light
Point Light
Directional Light
Spot Light

Ambient Light

- **Omnidirectional Constant Illumination**
  - Identical incident radiance from all directions

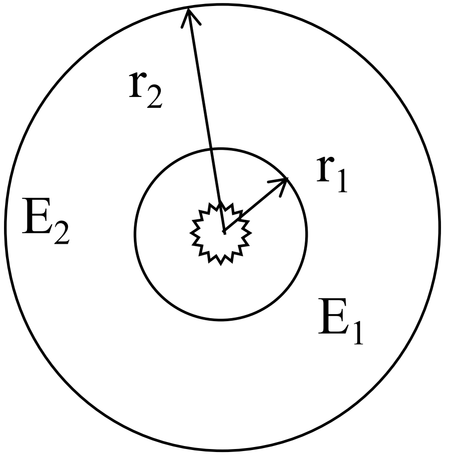
$$L_{rl}(x, \omega_o) = L_a \int_{\Omega_+} f_r(\omega_i, x, \omega_o) \cos\theta_i \, d\omega_i = L_a \, \rho_r(x, \omega_o)$$

- **Not Physically Plausible**
  - Crude approximation to indirect illumination

Point Light

- **Sphere of Radius *r***
  - Surface area: $4 \pi r^2$
- **Irradiance on Surrounding Sphere**
  - $E_r = \Phi_e / (4 \pi r^2)$
- **Quadratic Surface Area**
  - Double distance from emitter: sphere area four times bigger
- **Inverse Square Law**
  - Irradiance falls off with inverse of squared distance

$$\frac{E_1}{E_2} = \frac{r_2^2}{r_1^2}$$



Isotropic Point Light

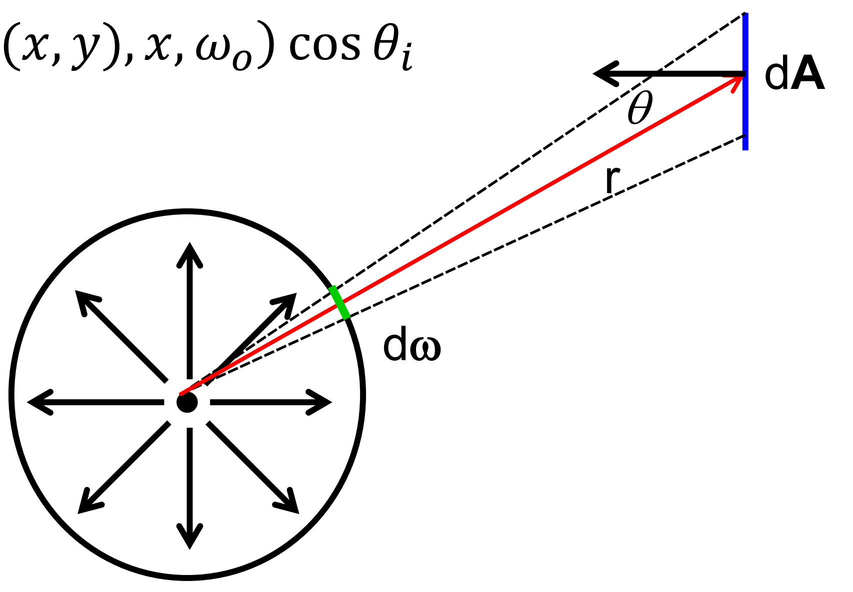
- **Emitted Intensity**
  - $I = \frac{\Phi_e}{4\pi}$
- **Irradiance on Surface dA**

$$E(x) = \frac{d\Phi_e}{dA} = \frac{d\Phi_e}{d\omega}\frac{d\omega}{dA} = I\frac{d\omega}{dA} = I\frac{dA\cos\theta}{r^2 dA} = I\frac{\cos\theta}{r^2}$$

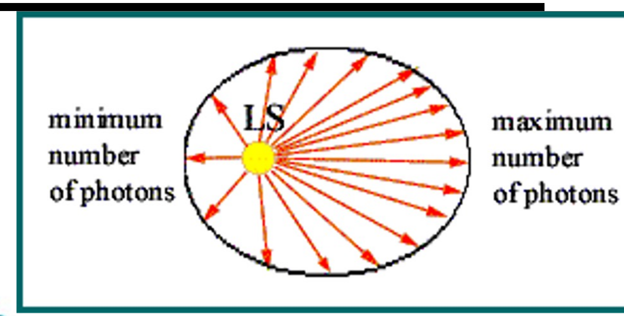
- **Illumination**

$$L_{rl}(x,\omega_o) = \frac{I}{\|x-y\|^2}V(x,y)f_r(\omega(x,y),x,\omega_o)\cos\theta_i$$

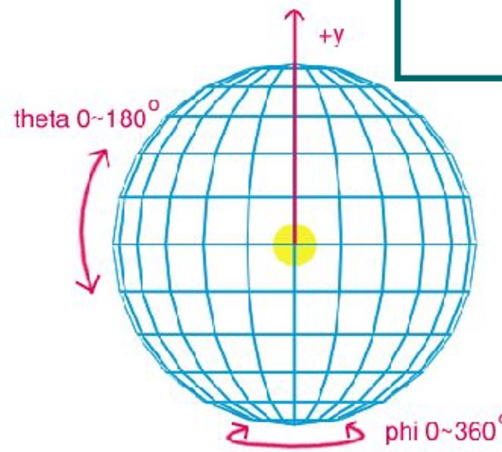
- **Extrinsic Parameters**
  - Position



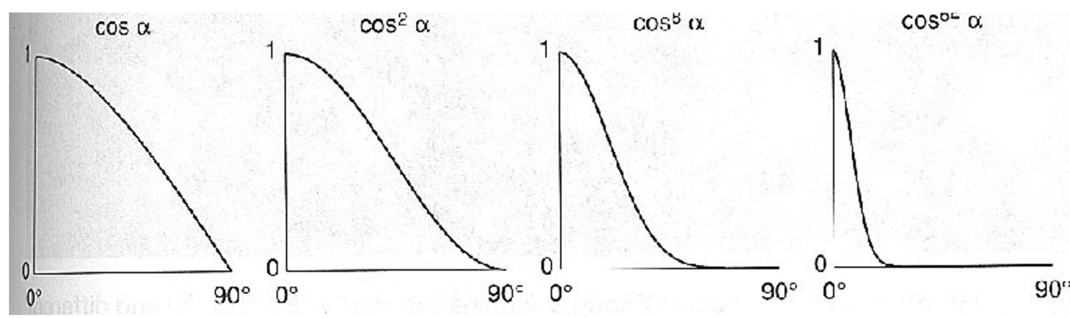
Anisotropic Point Light



- **Emitted Intensity**
  - $I(\omega) = \Phi_e P(\omega)$
- **Directional Distribution**
  - Tabulated
    - Goniometric diagram
  - Analytical
    - E.g. Warn (un-normalized)
      - Zero if dot product < 0
      - $P(\omega) = (\omega \cdot \omega_l)^n$



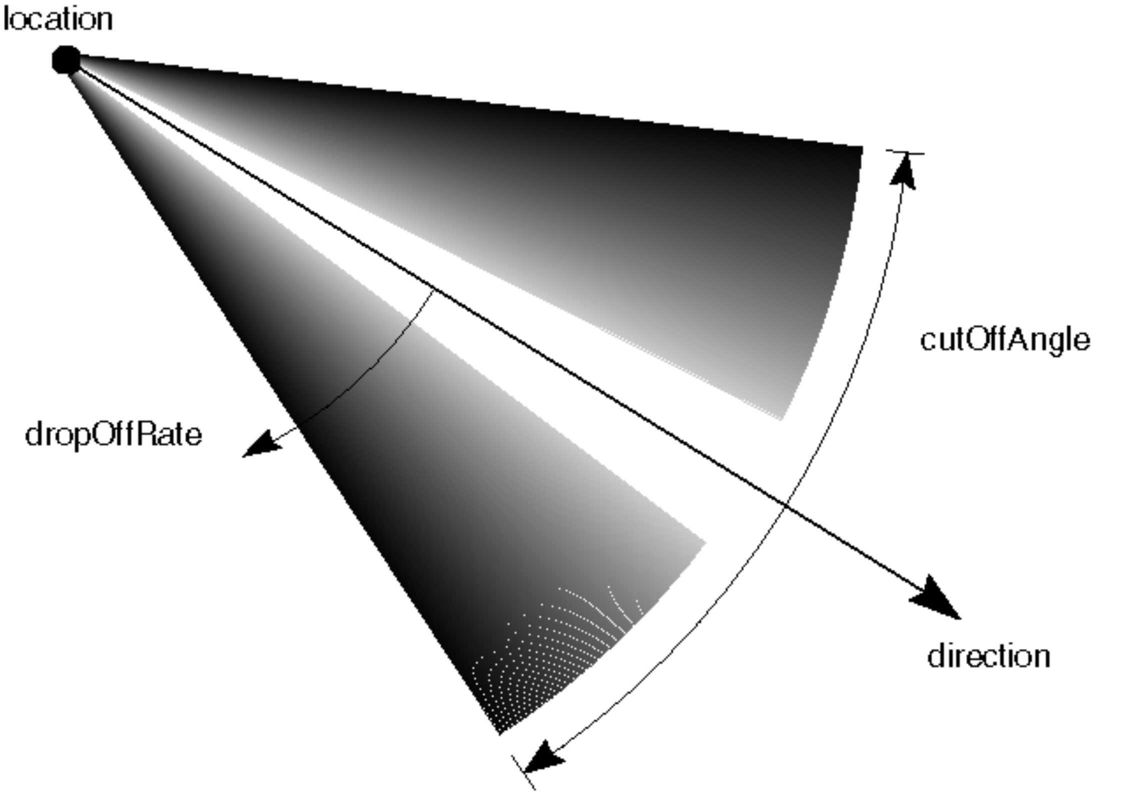
- **Extrinsic Parameters**
  - Position
  - Forward vector ω_l
- **Illumination**



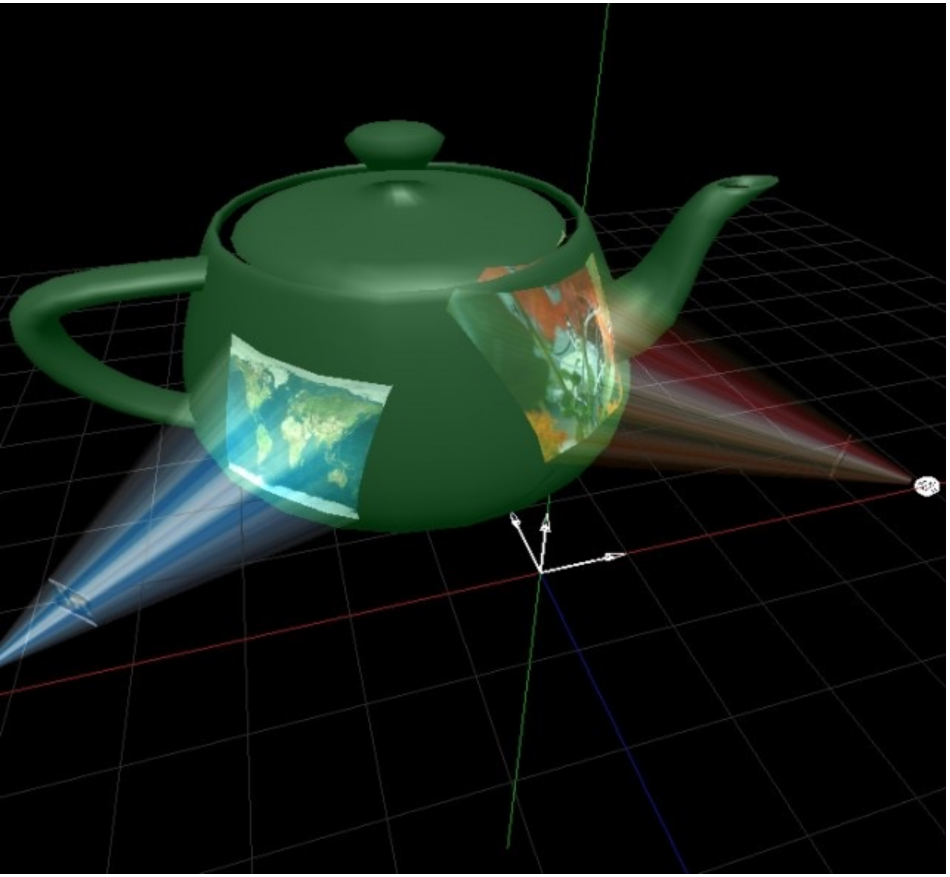
$$L_{rl}(x, \omega_o) = \frac{I(-\omega)}{\|x - y\|^2} V(x, y) f_r(\omega(x, y), x, \omega_o) \cos\theta_i$$

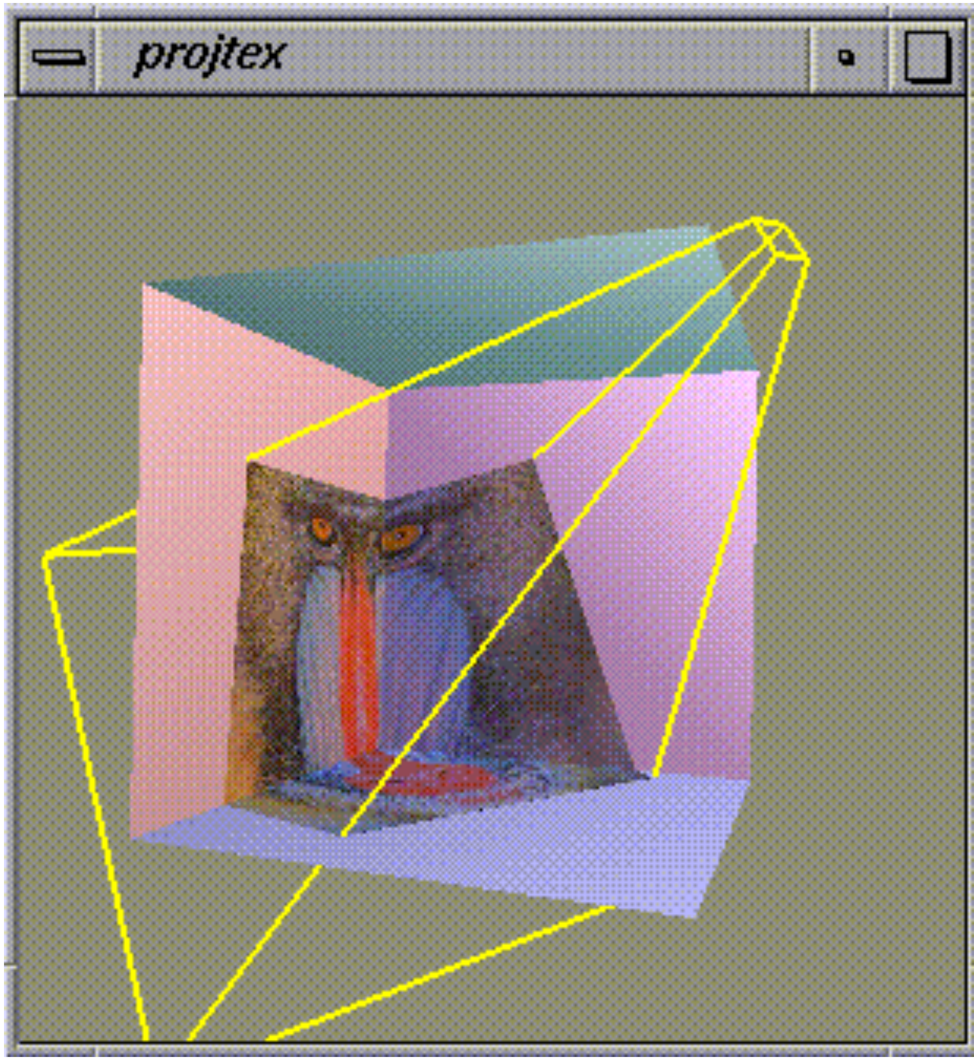
Spot Light

- **Restricted Directional Distribution**
  - If $\angle \omega, \omega_l < \theta_c \; then \; P(\omega) = (\omega \cdot \omega_l)^n$
  - Else $P(\omega) = 0$
  - With cut-off angle θ_c



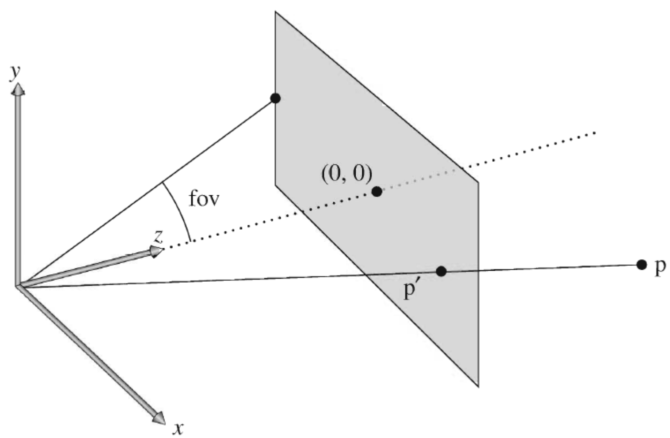
Projective Light

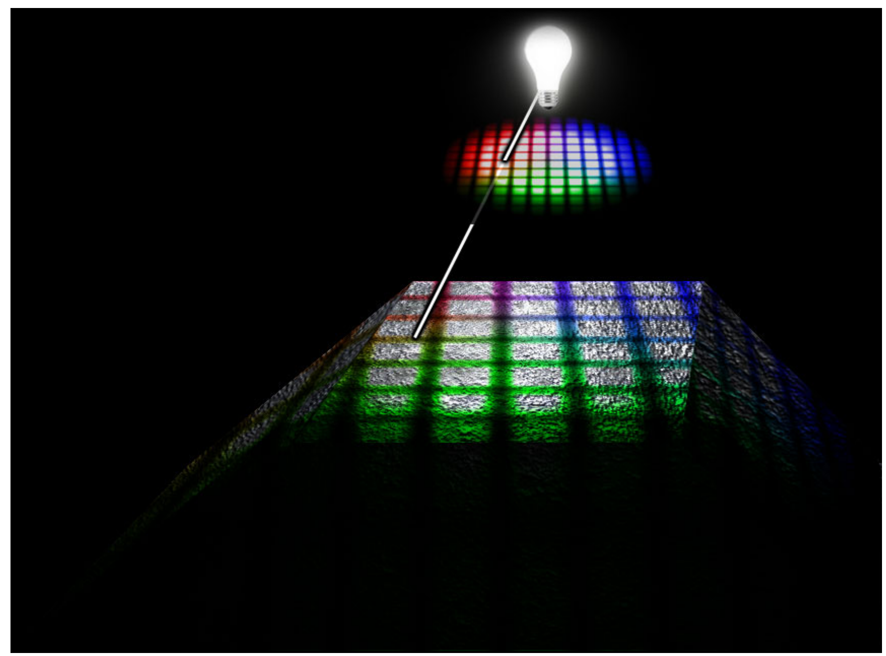




Projective Light

- **Unit direction from light center to surface point**
- **Find light-screen coordinates from ray direction**
  - Light-space coords: dot product with light basis vectors
  - Like for a perspective camera, but in reverse
- $P(\omega)$ **= color/intensity at corresponding coordinates**
- **Extrinsic Parameters**
  - Position
  - Forward vector
  - Up vector





Projective Light

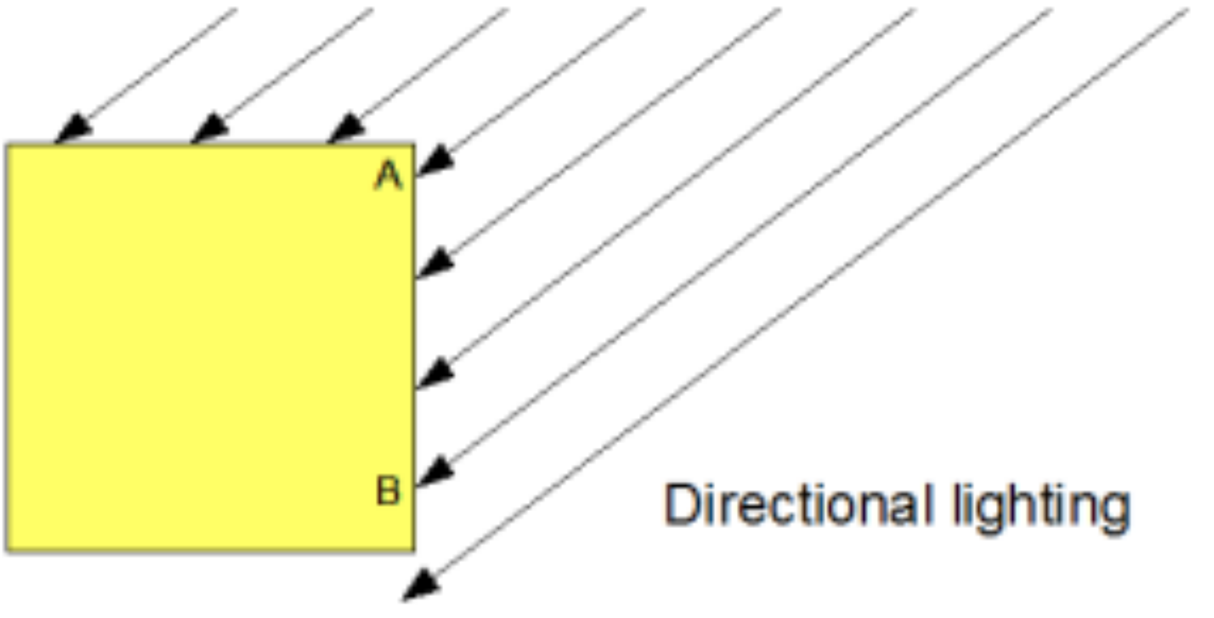
- **Examples**

Directional Light

- **Set point light to infinity**
  - In the limit, all light rays have parallel directions
- **Illumination**

$$L_{rl}(x,\omega_o) = B(y)V(x,\omega_i)f_r(\omega_i,x,\omega_o)\cos\theta_i$$

- **Extrinsic Parameters**
  - Forward vector



Sky Light

- **Sun**
  - Point source (approx.)
  - White light (by def.)
- **Sky**
  - Area source
  - Scattering: blue
- **Horizon**
  - Brighter
  - Haze: whitish
- **Overcast sky**
  - Multiple scattering in clouds
  - Uniform grey

Courtesy Lynch & Livingston

PRACTICAL APPROXIMATION

Rendering Equation: Approximations

$$L_o(x,\omega_o) = L_e(x,\omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

- **Approximations based only on empirical foundations**
  - An example: polygon rendering in OpenGL
- **Using RGB instead of full spectrum**
  - Follows roughly the eye's sensitivity
- **Sampling hemisphere along finite, discrete directions**
  - Simplifies integration to summation
- **Reflection function model**
  - Parameterized function
    - Ambient: constant, non-directional, background light
    - Diffuse: light reflected uniformly in all directions
    - Specular: light of higher intensity in mirror-reflection direction

Wrap Up

- **Physical Quantities in Rendering**
  - Radiance
  - Radiosity
  - Irradiance
  - Intensity
- **Light Perception**
- **Light Sources**
- **Rendering Equation**
  - Integral equation
  - Balance of radiance