Realistic Image Synthesis

- BRDFs and Direct Lighting -

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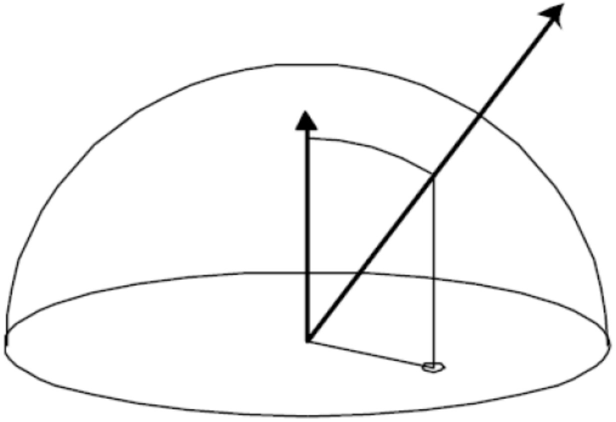
Importance Sampling Example

- **Example: Generate Cosine weighted distribution**
  - Generate ray according to cosine distribution with respect to normal
  - Need only average of the incident radiance

$$E = \int_{\Omega_i} L_i(\omega_i)\cos\theta_i d\omega_i = \int_0^{2\pi}\int_0^{\pi/2} L_i(\omega_i)\cos\theta_i \sin\theta_i d\theta_i d\phi_i$$

$$p(x) = \frac{\cos\theta_i \sin\theta_i}{\pi} \quad \Rightarrow \quad F_N = \frac{\pi}{N}\sum L_i(\xi_i)$$



Multidimensional Inversion

- **Multidimensional Inversion Method (here 2D)**
  - Goal: density function $p(x, y)$ with $x \in [a, b], y \in [c, d]$
  - Compute cumulative distribution function

$$P(x, y) = \int_{c}^{y} \int_{a}^{x} p(x', y') dx' dy'$$

  - Random variables along x are generated by integrating P over entire y-range (marginal density)

$$\xi_1 = P(\xi_1', y)\Big|_{y=d} = \hat{P}(\xi_1')$$

$$\Rightarrow \boxed{\xi_1' = \hat{P}^{-1}(\xi_1)}$$

  - Now, given $x = \xi'_1$ we have a one-dimensional problem but we still need to normalize

$$\xi_2 \sim P(x, \xi_2')\Big|_{x=\xi_1'} = \tilde{P}(\xi_2')$$

$$\Rightarrow \xi_2' \sim \tilde{P}^{-1}(\xi_2) \Rightarrow \boxed{\xi_2' = \frac{\tilde{P}^{-1}(\xi_2)}{\tilde{P}(d)}}$$

Multidim. Inversion: Hemisphere

- **Multidimensional probability function ($p(\omega) = const = c$)**

$$\int_{\Omega_+} \mathrm{p}(\omega) d\omega = 1 \quad \Rightarrow \quad c \int_{\Omega_+} d\omega = 1 \quad \Rightarrow \quad c = \frac{1}{2\pi}$$

$$p(\omega) = \frac{1}{2\pi} \quad \Rightarrow \quad p(\theta, \varphi) = \frac{\sin(\theta)}{2\pi} \;\; \text{with} \;\; d\omega = sin\theta \; d\theta \mathrm{d}\varphi$$

- **Marginal density function (integrating out φ)**

$$p(\theta) = \int_0^{2\pi} \frac{sin\theta}{2\pi} d\varphi = sin\theta \quad \Rightarrow \quad \xi_1 = P(\theta) = \int_0^{\theta} sin\theta' d\theta' = 1 - cos\theta$$

- **Conditional density function**

$$p(\varphi|\theta) = \frac{p(\theta, \varphi)}{p(\theta)} = \frac{1}{2\pi} \quad \Rightarrow \quad \xi_2 = P(\varphi|\theta) = \int_0^{\varphi} \frac{1}{2\pi} d\varphi' = \frac{\varphi}{2\pi}$$

- **Inverting, with: if $(1 - X)$ is uniform in $[0, 1]$, so is X**

$$\boxed{\theta = cos^{-1}(1 - \xi_1) = cos^{-1}\xi_1} \qquad \boxed{\varphi = 2\pi\xi_2}$$

Sampling a BRDF

- **Uniformly distributed on the hemisphere ($\sim d\omega$)**

$$\varphi = 2\pi\xi_1$$
$$\theta = \mathrm{acos}(\xi_2)$$

$$x = \cos(2\pi\xi_1)\sqrt{(1-\xi_2^2)}$$
$$y = \sin(2\pi\xi_1)\sqrt{(1-\xi_2^2)}$$
$$z = \xi_2$$

- **Cosine distributed on the hemisphere ($\sim cos\theta \mathrm{d}\omega$)**

$$\varphi = 2\pi\xi_1$$
$$\theta = \mathrm{acos}(\sqrt{\xi_2})$$

$$x = \cos(2\pi\xi_1)\sqrt{(1-\xi_2)}$$
$$y = \sin(2\pi\xi_1)\sqrt{(1-\xi_2)}$$
$$z = \sqrt{\xi_2}$$

- **Cosine-power distributed on the hemisphere ($\sim cos^n\theta d\omega$)**

$$\varphi = 2\pi\xi_1$$
$$\theta = \mathrm{acos}(\xi_2^{1/(n+1)})$$

$$x = \cos(2\pi\xi_1)\sqrt{(1-\xi_2^{2/(n+1)})}$$
$$y = \sin(2\pi\xi_1)\sqrt{(1-\xi_2^{2/(n+1)})}$$
$$z = \xi_2^{1/(n+1)}$$

Also see
Global Illumination Compentium
by Philip Dutre (U. Leuven):
http://www.cs.kuleuven.ac.be/~phil/GI/

Sampling a BRDF

- **Phong BRDF**

$$\int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i \cos\theta d\omega = k_d \int_{\Omega_+} L_i \cos\theta d\omega + k_s \int_{\Omega_+} L_i \cos^n \theta' \cos\theta d\omega$$

- **Sampling the diffuse part**
  - Uniform on the hemisphere
  - Better: Cosine distributed
- **Sampling the glossy part**
  - Uniform on the hemisphere
  - Better: Cosine distributed
  - Cosine-power distributed
    - Cdigon integrates only over positive hemisphere (see GI-Compendium)

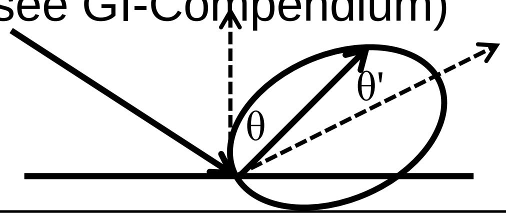
$$I_1 = \frac{2\pi k_d}{N} \sum_{k=1}^{N} L_k(\omega_k) \cos\theta_k$$

$$I_1 = \frac{\pi k_d}{N} \sum_{k=1}^{N} L_k(\omega_k)$$

$$I_2 = \frac{2\pi k_s}{N} \sum_{k=1}^{N} L_k(\omega_k) \cos^n \theta'_k \cos\theta_k$$

$$I_2 = \frac{\pi k_s}{N} \sum_{k=1}^{N} L_k(\omega_k) \cos^n \theta'_k$$

$$I_2 = \frac{\pi k_s}{N} C_{\text{digon}} \sum_{k=1}^{N} L_k(\omega_k) \cos\theta_k$$



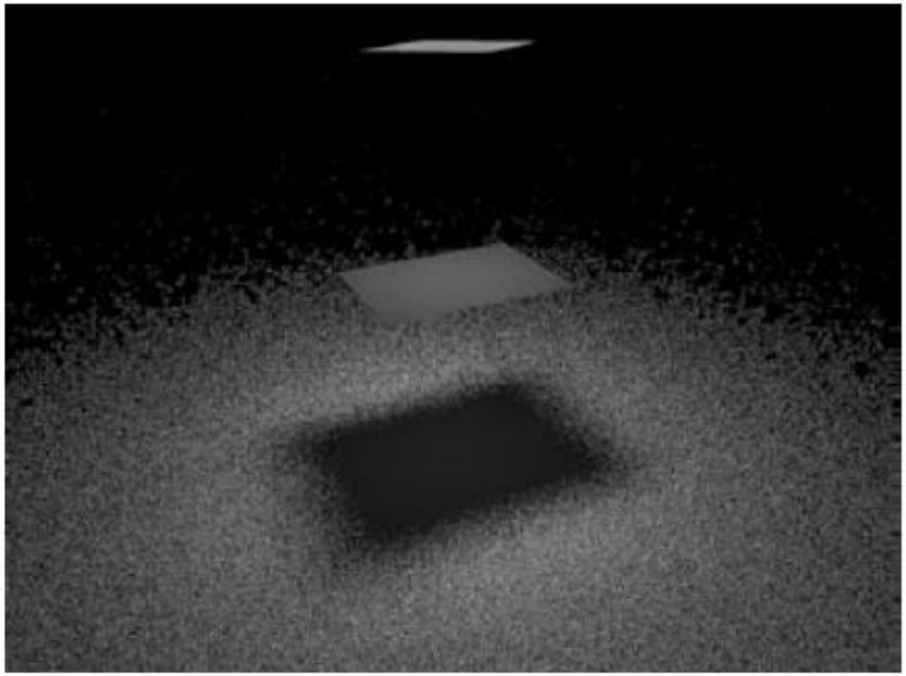
Direct Lighting Computation

- **Need to compute the integral**

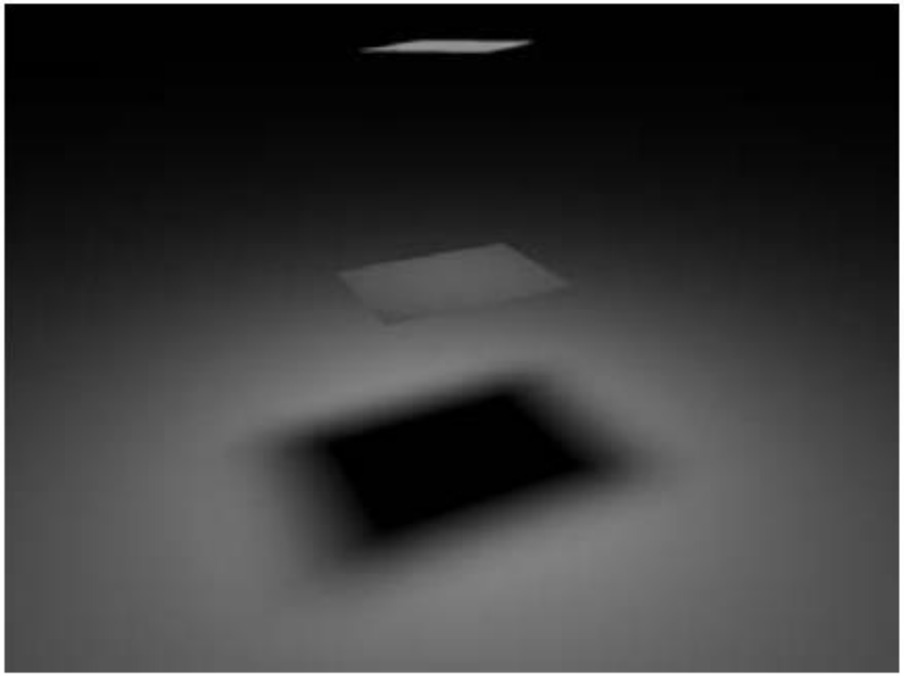
$$L(x,\omega_o)=L_e(x,\omega_o)+$$
$$\int_{y\in S} \underbrace{f_r(\omega_o,x,\omega_i)L_i(x,\omega_i)}_{\text{usually low variance if mostly diffuse}} \underbrace{V(x,y)}_{\text{unknown variance}} \underbrace{\frac{\cos(\theta_x)\cos(\theta_y)}{\|x-y\|^2}dA_y}_{\text{high variance}}$$

- **See also**
  - Shirley et al.: MC-Techniques for Direct Lighting Calculations
- **Single light source, not too close (>1/5 of its radius)**
  - Small:
    - $1/r^2$ has low variance, $\cos\theta_x$ has low variance
  - Planar:
    - $\cos\theta_y$ has low variance too
    - Choose samples uniformly on light source geometry
      - Sampling directions could have high variance
  - For curved light sources
    - Take into account orientation/normal

Direct Lighting Computation

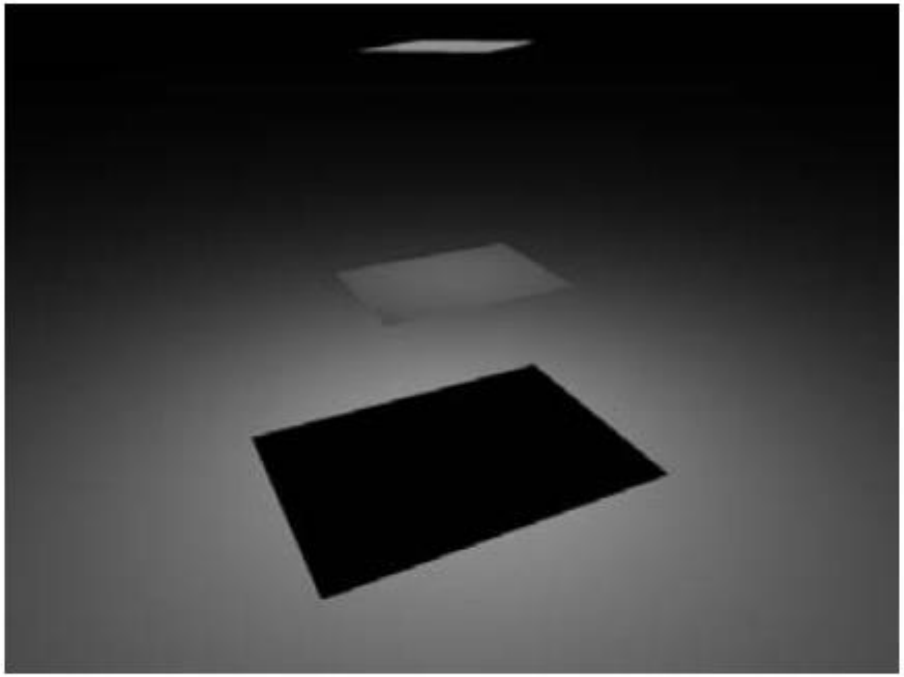


Sampling projected solid angle
4 eye rays per pixel
100 shadow rays

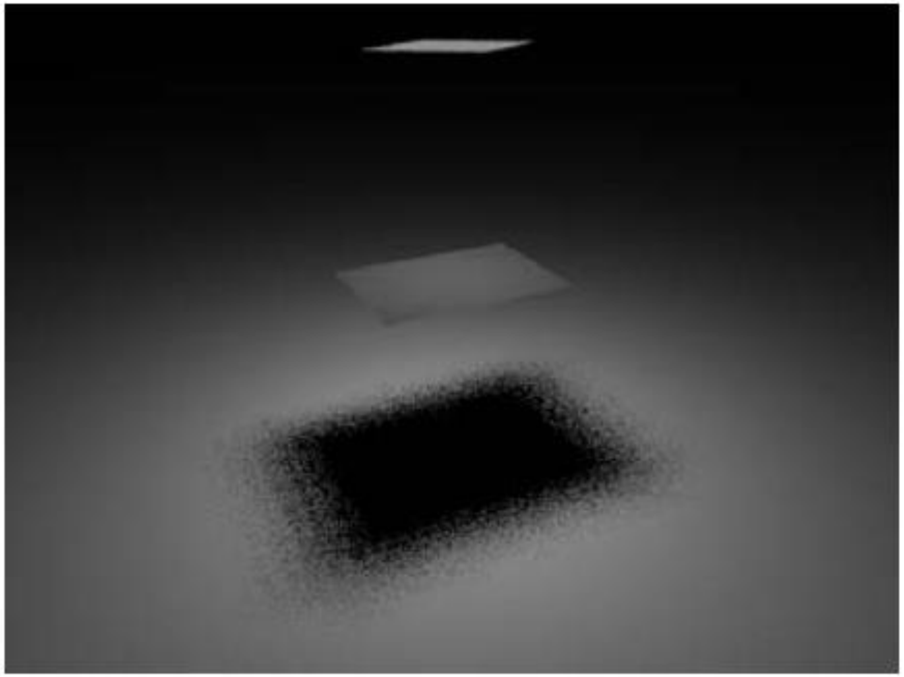


Sampling light source area
4 eye rays per pixel
100 shadow rays

Direct Lighting Computation

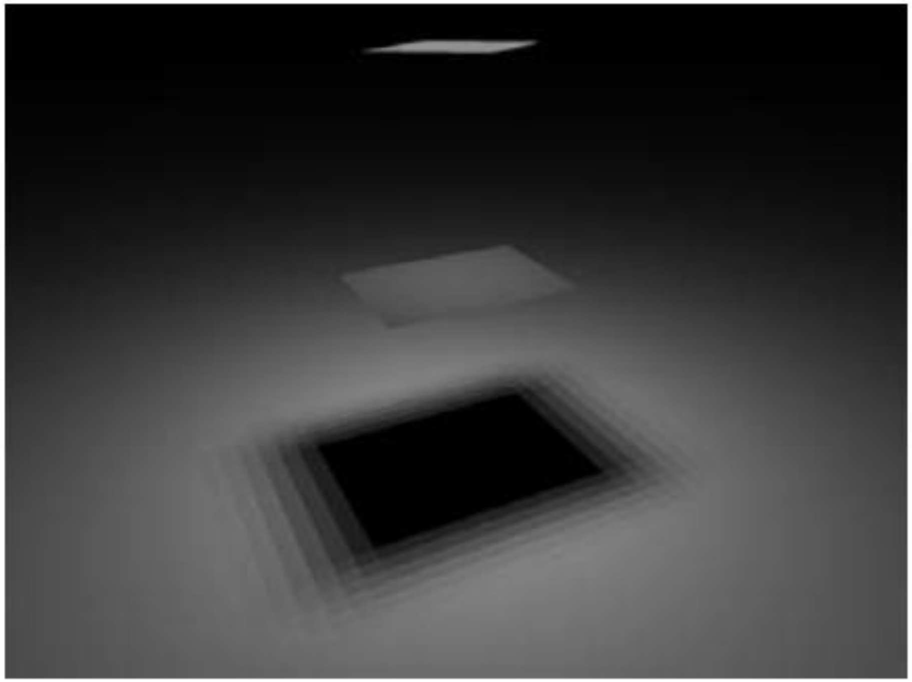


Fixed sample location
4 eye rays per pixel
1 shadow ray each

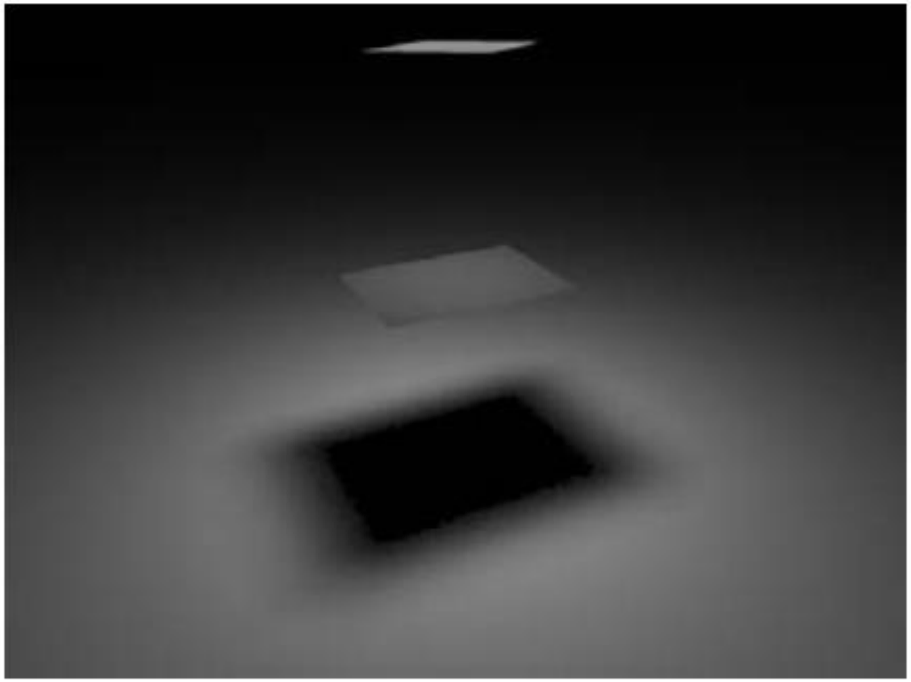


Random sample location
4 eye rays per pixel
1 shadow ray each

Direct Lighting Computation

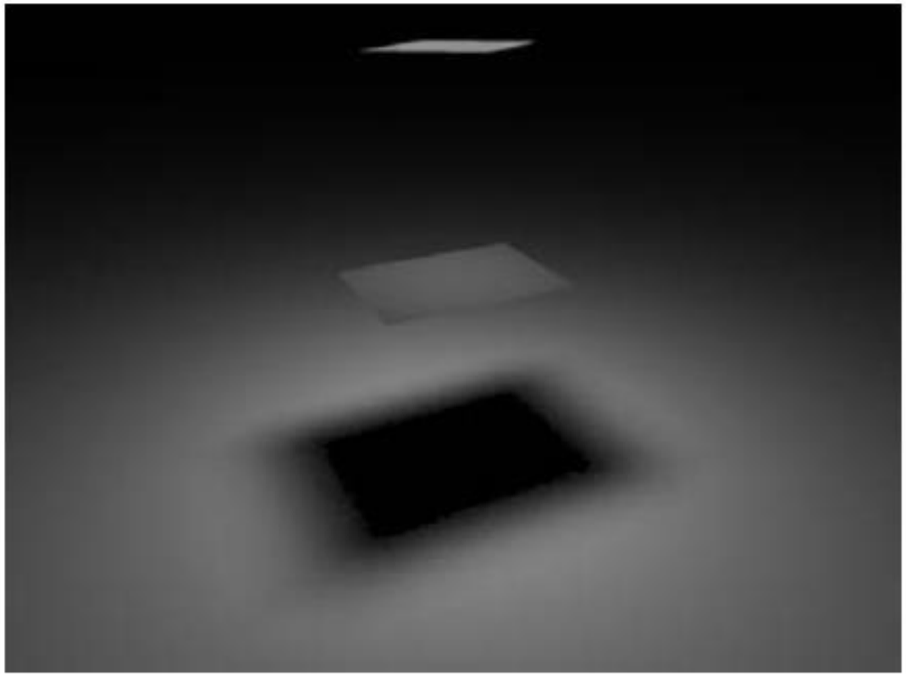


Sample locations on 2D grid
4 eye rays per pixel
64 shadow ray

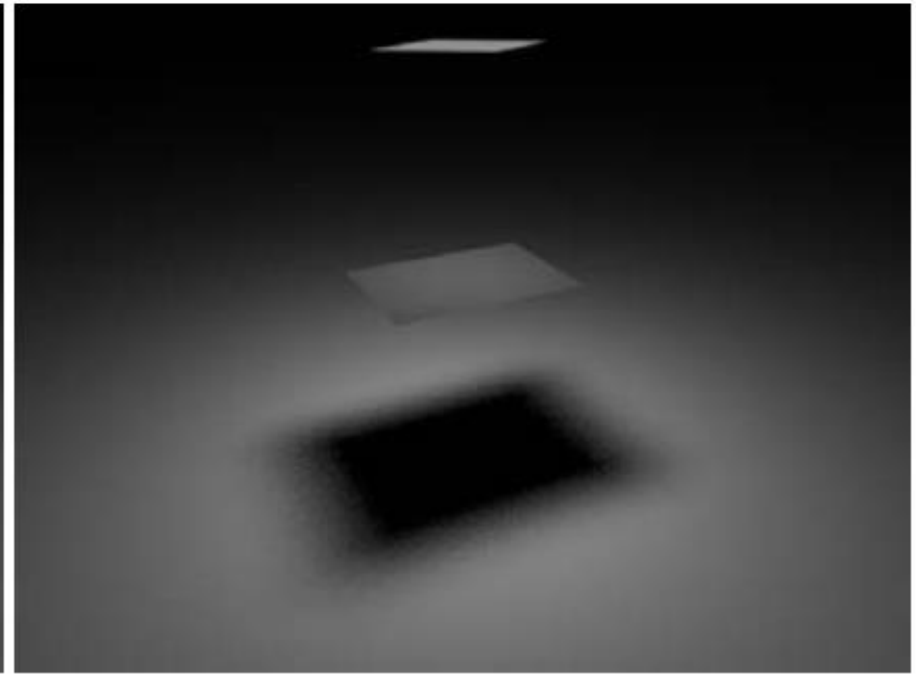


Stratified random sample locations
4 eye rays per pixel
64 shadow ray

Direct Lighting Computation



Stratified random sample locations
4 eye rays per pixel
16 shadow ray



Stratified random sample locations
64 eye rays per pixel
1 shadow ray

Direct Lighting Computation

- **Importance sampling of many light sources**
  - Ray tracing cost grows with number of lights
- **Approaches**
  - Equal probability ($1/N_L$)
  - Fixed weights according to total power of light
    - Sample as discrete probability density function
    - Make sure that pdf is not zero if light could be visible
      - Must use conservative approximation
  - Stratification through spatial subdivision
    - Estimate the contribution of lights in each cell (e.g. octree)
  - Dynamic and adaptive importance sampling
    - Compute a running average of irradiance at nearby points
    - Use the relative contribution as the importance function
    - Should use coherent sampling
    - Might need to estimate separately for primary and secondary rays

$$p_i = \frac{\Phi_i}{\sum \Phi_i}$$

Direct Lighting Computation

- **Example: Sampling thousands of lights interactively**
  - At each pixel send random path into the scene & towards some light
    - Low overhead since we already trace many rays per pixel
  - Gives a rough estimate of light contribution to the entire image
    - Take maximum contribution of each light at any pixel
    - Might want to average over several images (less variance)
  - Use this estimate for importance sampling
    - Make sure every light is sampled eventually
    - Might ignore lights with very low probability (but introduces bias)
  - Trace samples ONLY from the eye
    - Avoids touching the entire scene
    - Minimizes working set for very large scenes
  - Published as [Wald et al., Interactive Global Illumination in Complex and Highly Occluded Environments, EGSR'03]