# Realistic Image Synthesis

## Bidirectional Path Tracing & Reciprocity

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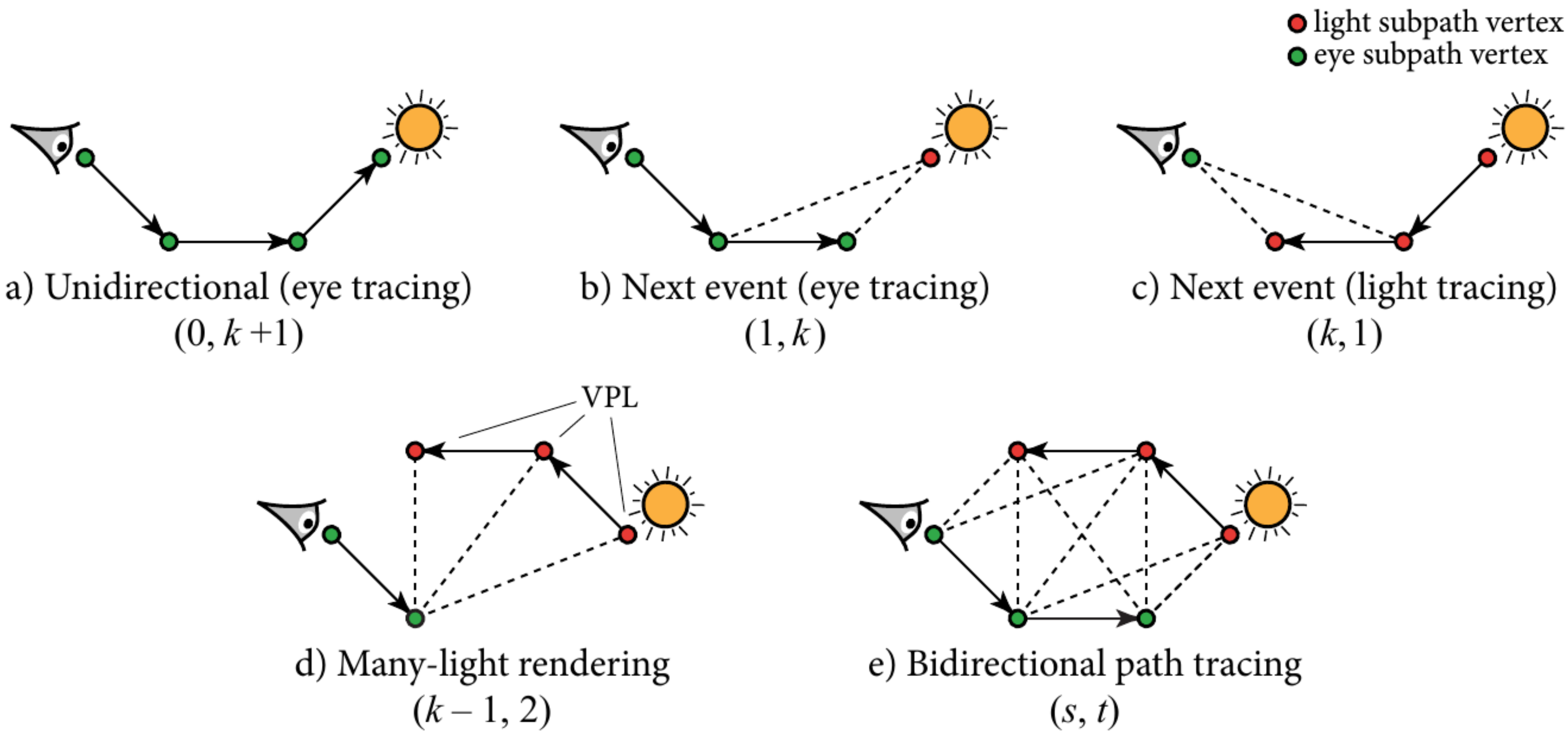
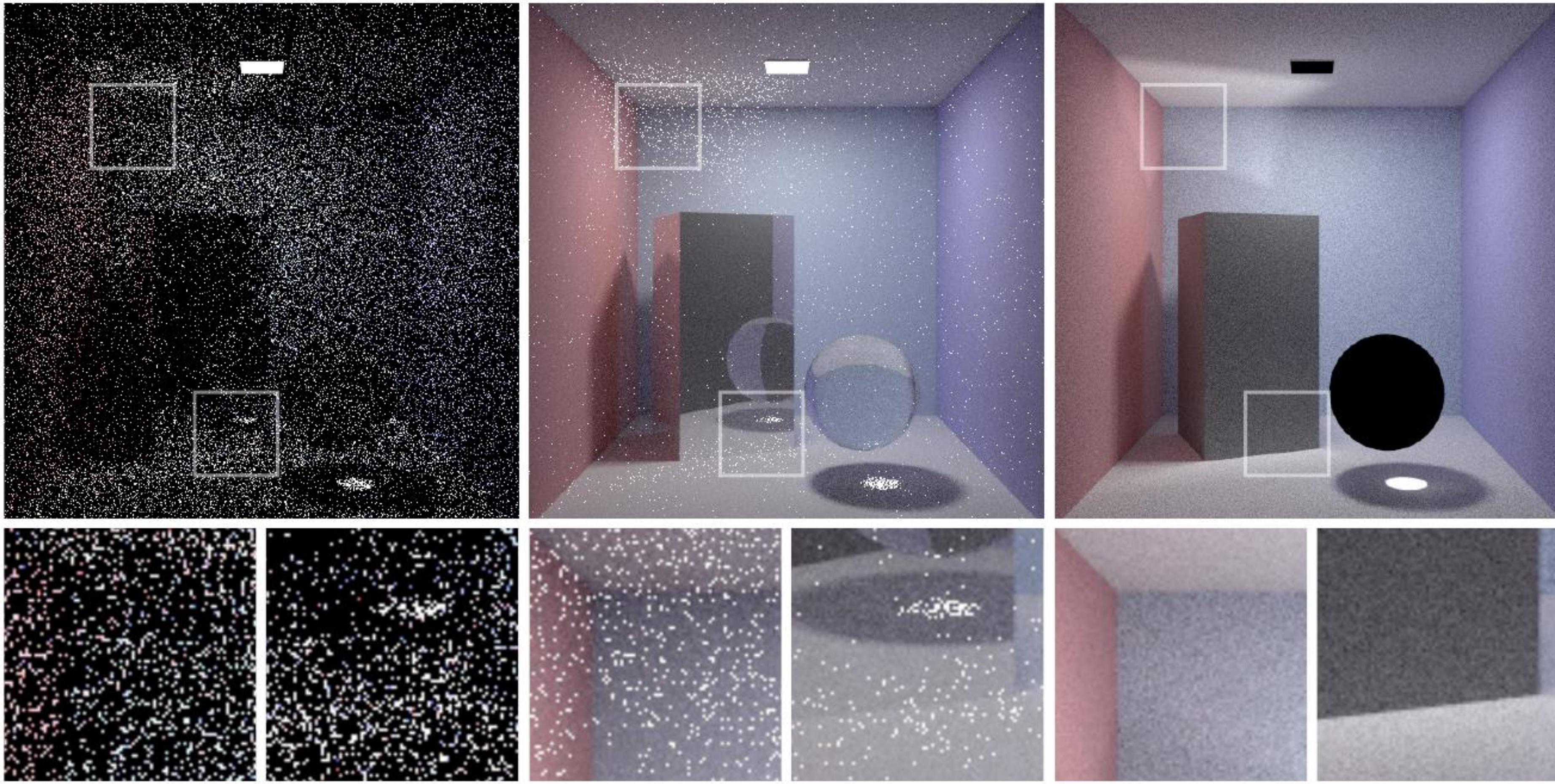
# Path Sampling Techniques




- **Different techniques of sampling paths from both sides**
  - Numbers in parenthesis are # of vertices traced from light/camera, resp.
  - See later, for many light methods

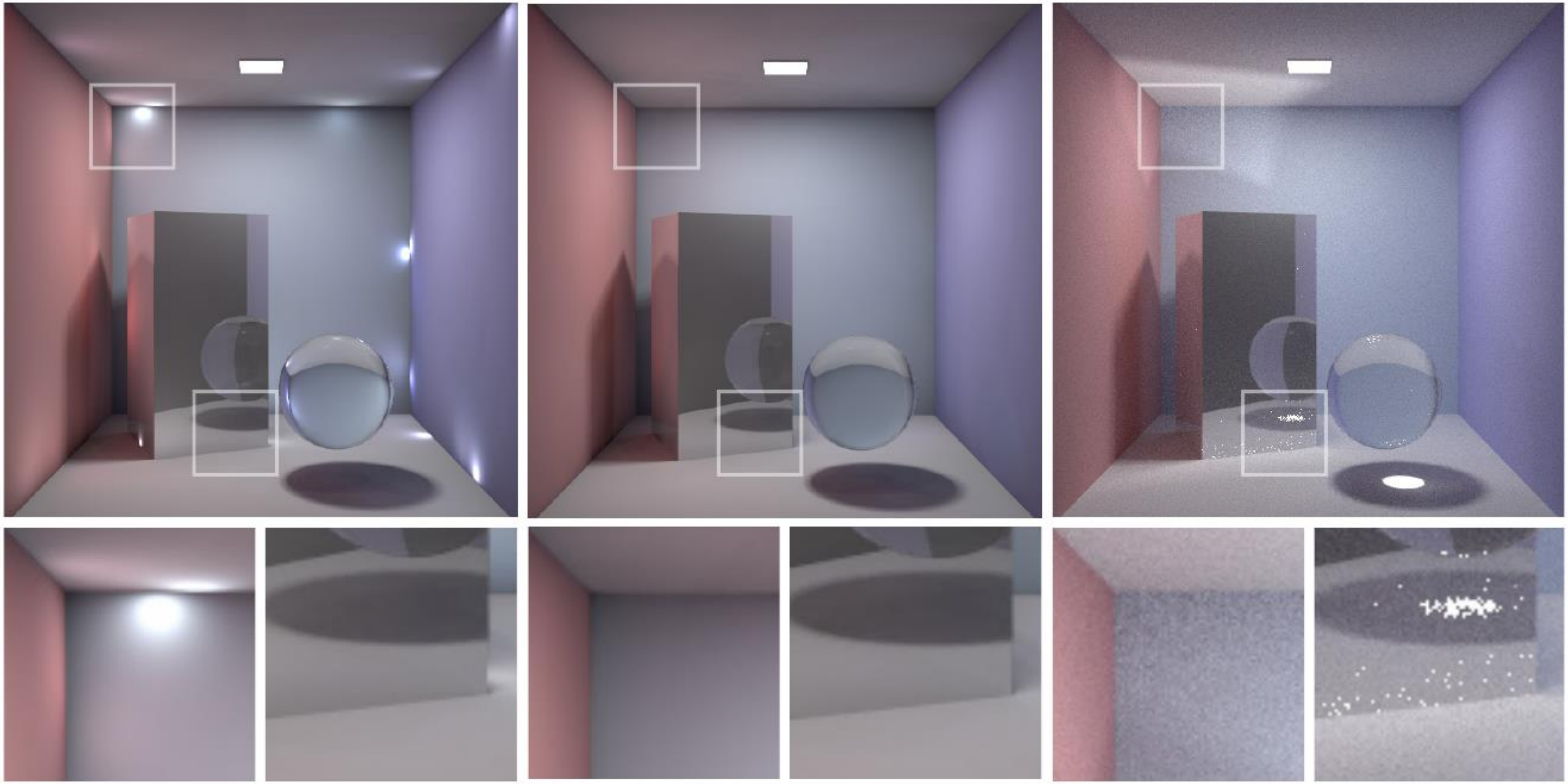
# Results from Different Techniques



a) Unidirectional (eye tracing) b) Unidirectional + next event c) Next event (light tracing)

- **Results from tracing 40 paths per pixel**

# Results from Different Techniques



d) Instant radiosity e) Instant radiosity (clamped) f) Bidirectional path tracing

- **Results from tracing 40 paths per pixel**
  - f): „Problem of insufficient techniques“ for sampling SDS paths

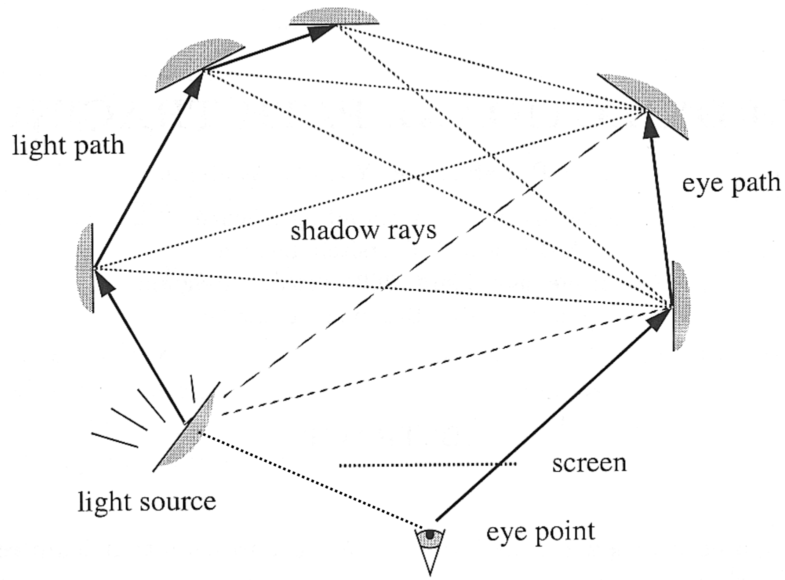
# BIDIRECTIONAL PATH TRACING

# Light & Path Tracing

- **Problem:**
  - Probability of hitting the camera from the light sources is almost zero
  - Probability of hitting the light source is often also very small
    - Next Event Estimator: Try to find a direct connections
      - Non-optimal (e.g. on mirror surface)
      - Ignores secondary light sources (e.g. via mirror, at caustics)
- **Approaches:**
  - Bidirectional Path Tracing
    - Combination of eye and light paths
    - Weighted MC sampling for best results
    - Includes Vertex Connection and Merging (VCM, later)
  - Metropolis-Sampling [Veach´1997] (see later)
    - Random variation and mutations of bidirectional paths
    - Very well suited for very complex light paths
    - Unbiased but relatively complex algorithms
    - Uneven convergence

# Bidirectional Path-Tracing

- **Idea: Combine Paths from Both Sides**
  - Generate path from the light sources and the camera
  - Connect paths deterministically (every pair of two hit points)
    - Different probabilities of generating paths
  - Compute weighted sum of contributions (→ MIS)
- **References:**
  - **Lafortune et al.**, Bidirectional Path-Tracing, [CompuGraphics`93]
  - **Veach, Guibas**, Bidirectional Estimators for LightTransport, [EGRW´94, Siggraph´95]



# Solving the Rendering Equation

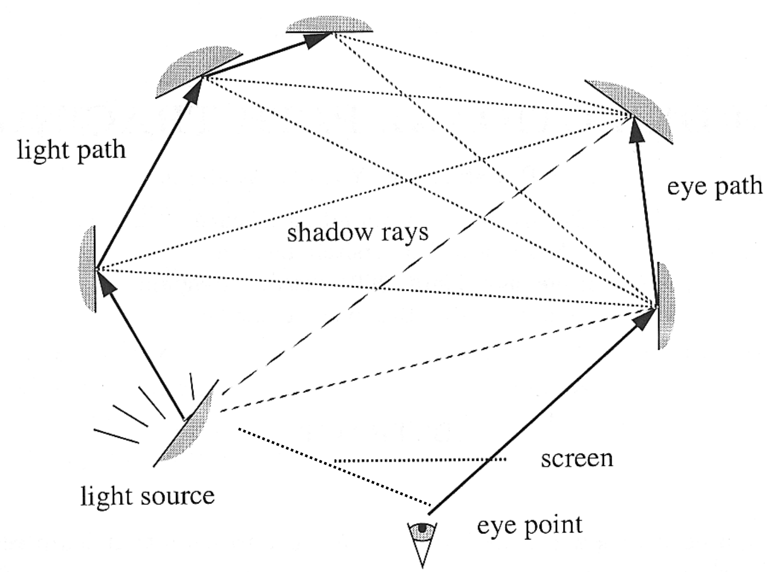
- **Von Neumann Expansion of Measurement Equation**

$$I_p = \int_{S\times S} L_e(x \to x')G(x \to x')W_p(x \to x')dA(x)dA(x') +$$

$$+ \int_{S\times S\times S} L_e(x \to x')G(x \to x')f_r(x \to x' \to x'')G(x' \to x'')\, W_p(x'$$

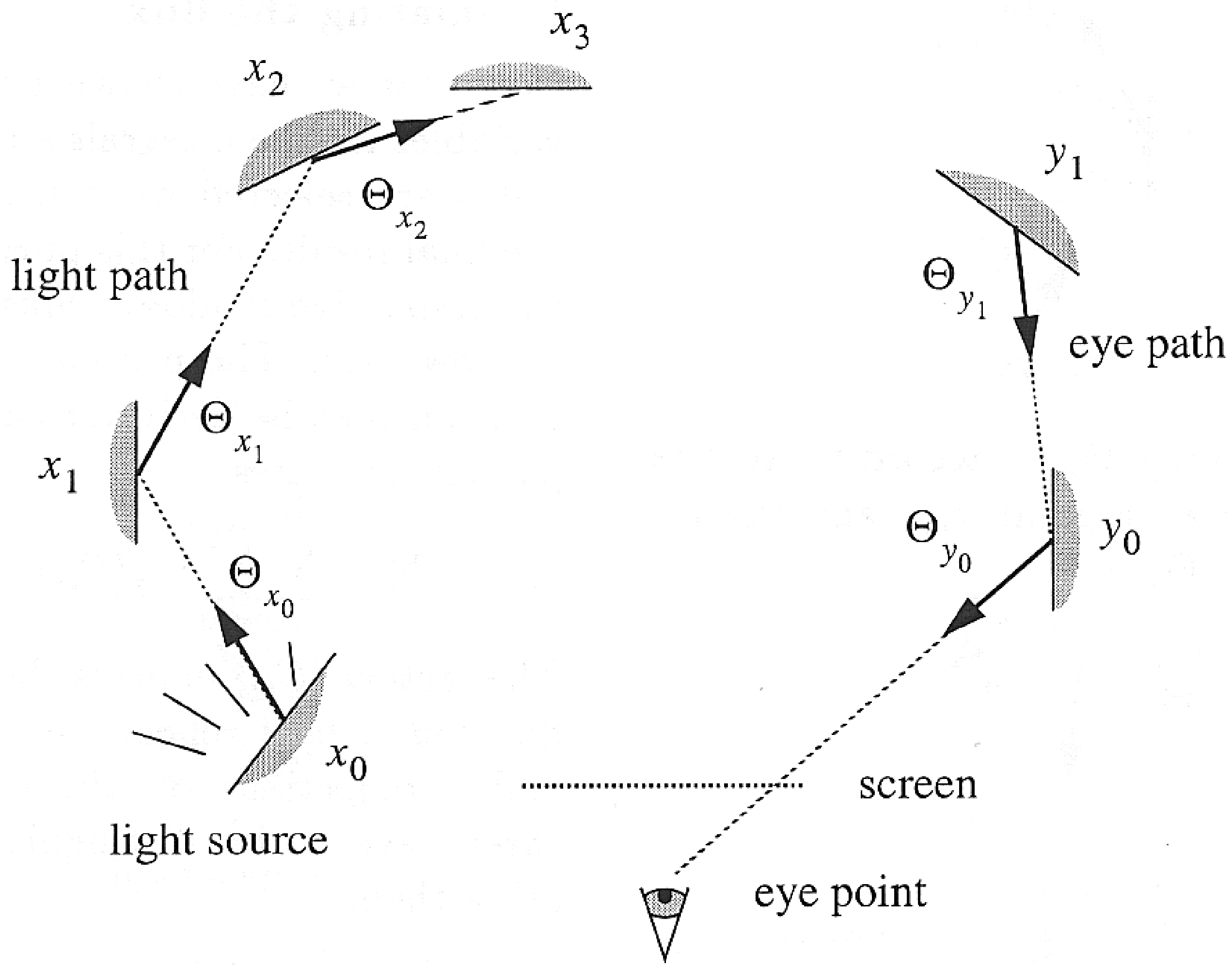
$$\to x'')dA(x)dA(x')dA(x'') + \cdots \qquad with\ G(x,y) = \frac{cos\theta_x cos\theta_y}{\|x-y\|^2}$$

  - Independent estimation of all paths with fixed lengths
  - Bidirectional generation of paths
  - Weighted MC integration for each term (MIS)
  - More efficient by reusing costly paths (i.e. visibility samples) multiple times
  - Typically: One pair of paths per pixel sample



# Bidirectional Path-Tracing

- **Notation**

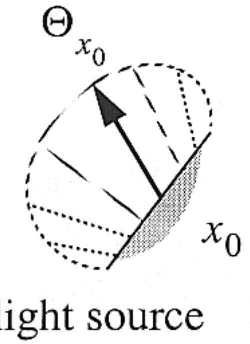


# Bidirectional Path-Tracing

- **Generating Light Paths (example)**
  - On the light source

$$p(x,\Theta_x) = \frac{L_e(x,\Theta_x)\,\|\Theta_x \cdot N_x\|}{\Phi}$$

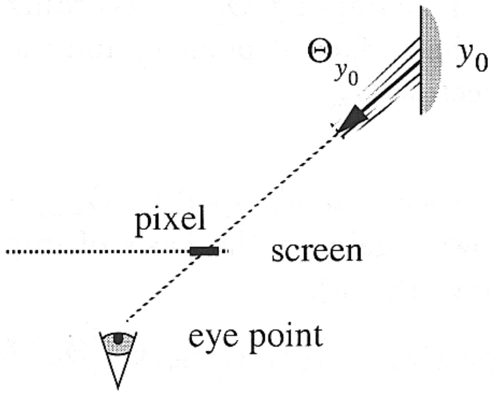
$$\Phi = \int_A \int_{\Omega_+} L_e(x,\Theta_x)\,\|\Theta_x \cdot N_x\|\,d\Theta_x dA_x$$




- **Generating Eye Paths (example)**
  - On the eye/camera (via point in the scene)

$$p(y,\Theta_y) = \frac{g(y,\Theta_y)W(y,\Theta_y)\,\|\Theta_y \cdot N_y\|}{G}$$

$$G = \int_A \int_{\Omega_+} g(y,\Theta_y)W(y,\Theta_y)\,\|\Theta_y \cdot N_y\|\,d\Theta_y dA_y$$

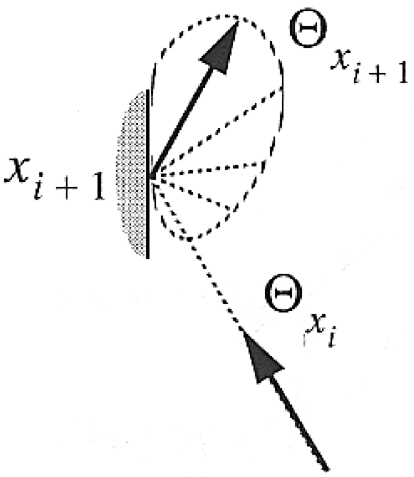



  - g(): 1, if point is visible in this direction

# Bidirectional Path-Tracing

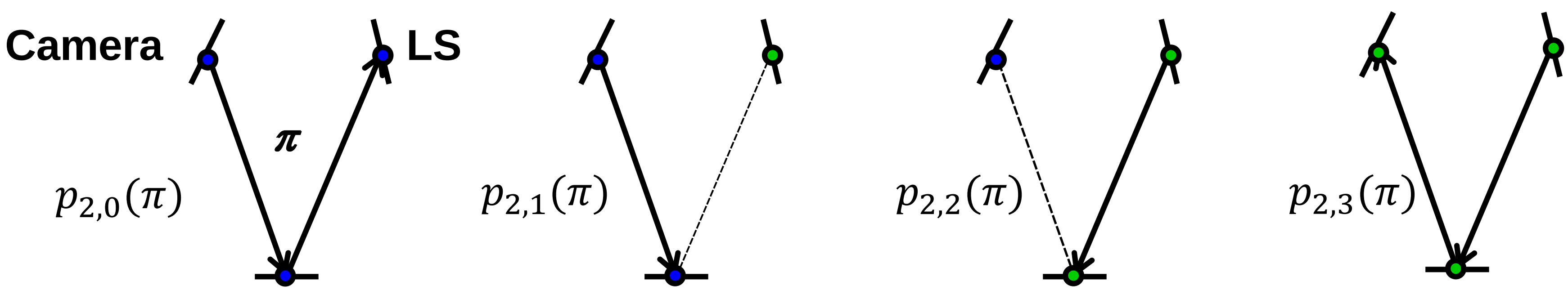
- **Extension of Paths at Hit Points**
  - Identical for both directions
    - Reciprocity of BRDF under reflection
  - Use whatever BRDF sampling technique suits best
    - But must be a joint probability (conditioned on the previous point)
      - This does include uniform probability on any surface
      - (But not a point generated from some other point, e.g. due to occl.)
    - E.g.

$$p(\Theta) = f_r(\Theta_{x_i}, x_{i+1}, \Theta) \, | \Theta_{x_{i+1}} \cdot N_{x_{i+1}} |$$



# Bidirectional Probabilities

- **Probabilities of Paths $\pi$ in Bidirectional Path Tracing**
  - Different locations of *vertex connections* (see VCM later)
  - k : length of paths (# of transports or segments)
  - m: # of vertices generated from light source
    - 0: None
    - 1: Vertex on light source
    - 2: Vertex on light source and directional sample
    - Etc.
  - Similar for paths from the eye
  - $p_{k,m}(\pi)$ : Probability to choose path $\pi$ with method (k,m)

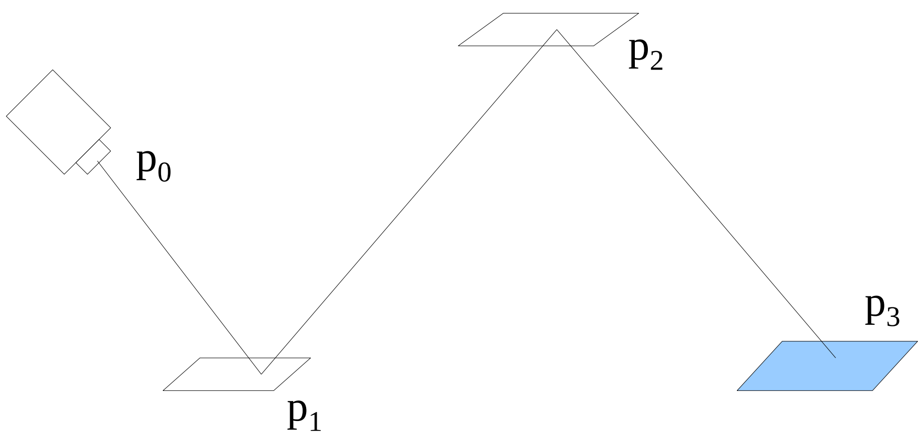


# Mathematical Formulation

- **Rendering Equation with Area Parametrization**

$$
\begin{aligned}
L(p_1 \to p_0) = {} & L_e(p_1 \to p_0) + \\
& \int_A L_e(p_2 \to p_1) f(p_2 \to p_1 \to p_0) G(p_2 \to p_1) \, dA(p_2) + \\
& \int_A \int_A L_e(p_3 \to p_2) f(p_3 \to p_2 \to p_1) G(p_3 \to p_2) \\
& f(p_2 \to p_1 \to p_0) G(p_2 \to p_1) \, dA(p_3) \, dA(p_2) + \cdots
\end{aligned}
$$

$$
with \quad G(p_2 \to p_1) = V(p_1, p_2) \frac{\cos(\theta_{p_1}) \cos(\theta_{p_2})}{|p_2 - p_1|^2}
$$



# Mathematical Formulation

- **Path Formulation**
  - $\pi_i$: Path of length i $\quad L(p_1 \to p_0) = \sum_{i=1}^{\infty} L(\pi_i(p_1, p_0)) = \sum_{i=1}^{\infty} L(\pi_i)$

$$L(\pi_i) = \underbrace{\int_A \int_A \cdots \int_A}_{i-1} L_e(p_i \to p_{i-1}) \left( \prod_{j=1}^{i-1} G(p_{j+1} \to p_j) f(p_{j+1} \to p_j \to p_{j-1}) \right) dA(p_2) \cdots dA(p_i)$$

- ***Connection Throughput* T(π) of a path π**

$$T(\pi_i) = \prod_{j=1}^{i-1} G(p_{j+1} \to p_j) f(p_{j+1} \to p_j \to p_{j-1})$$

$$L(\pi_i) = \underbrace{\int_A \int_A \cdots \int_A}_{i-1} L_e(p_i \to p_{i-1}) T(\pi_i) dA(p_2) \cdots dA(p_i)$$

- **With Measurement**

$$I = \int_A \int_{A_{\text{pixel}}} L(p_1 \to p_0) G(p_1 \to p_0) W(p_1 \to p_0) dA(p_0) dA(p_1)$$

$$I = \sum_i \underbrace{\int_A \int_A \cdots \int_A}_{i+1} L_e(p_i \to p_{i-1}) T(\pi_i) G(p_1 \to p_0) W(p_1 \to p_0) dA(p_0) \cdots dA(p_i)$$

# Mathematical Formulation

- **Path Tracing with Russian Roulette**

$$L(p_1 \rightarrow p_0) = \sum_{i=1}^{\infty} L(\pi_i) = L(\pi_1) + \frac{1}{1-q_2} \sum_{i=2}^{\infty} L(\pi_i)$$

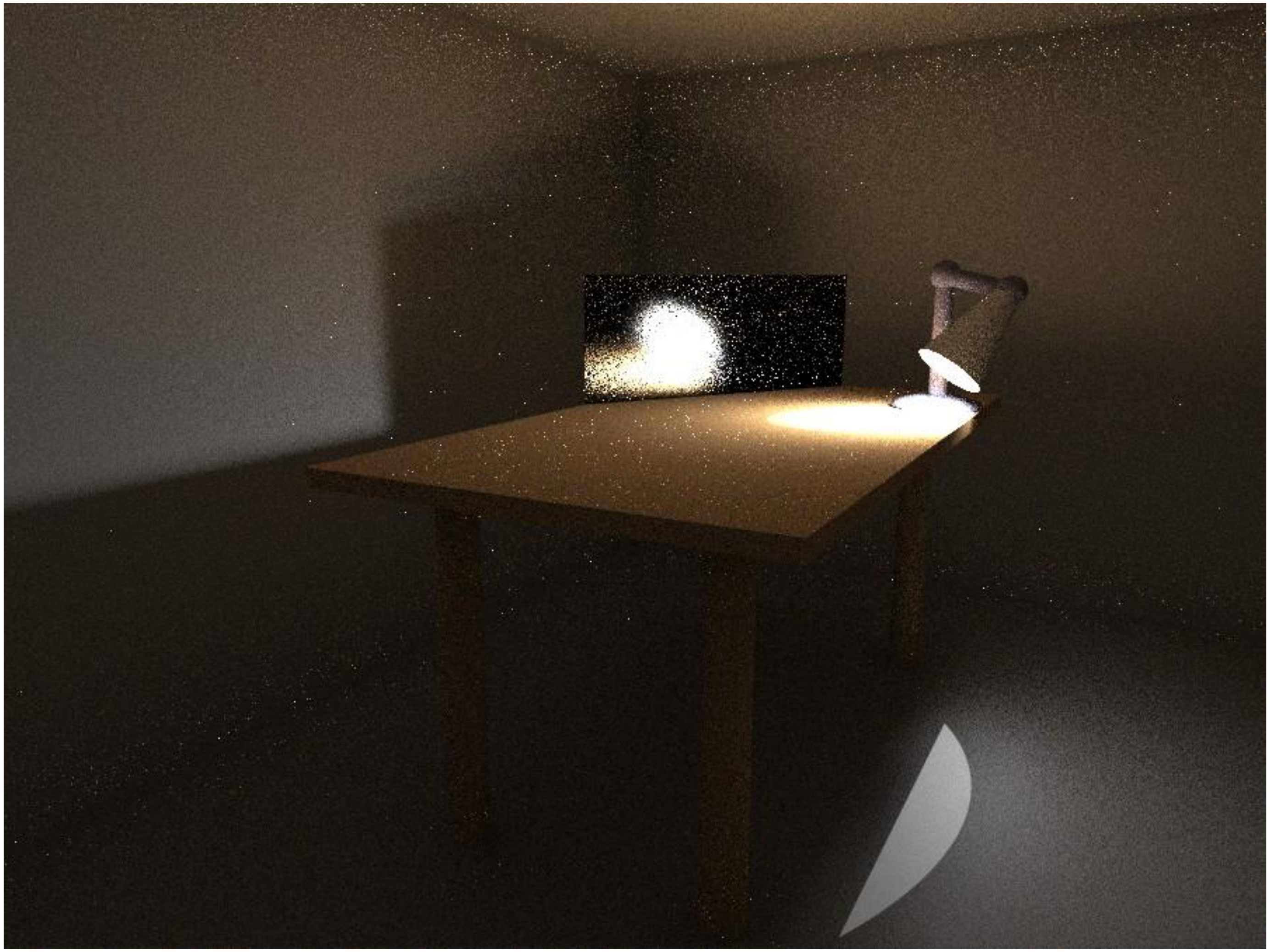
  – And similar for higher path lengths

- **How to choose the probabilities of sample points**
  – Whatever works, from wherever (!!!), e.g.
    - Area (uniform): $p_A(p_i) = \frac{1}{\sum_j A_j}$
    - Solid angle, depending on direction from previous sample: $p_A = p_\omega \frac{\cos\theta_i}{r^2}$
    - Any other joint probability that integrates to one over all surfaces and is non-zero where there could be a contribution
    - Must be a conditional probability, based on the previous point

- **Splitting of BRDFs or Emissions**
  – Make sure all path are accounted for !
  – Make sure no path is counted multiple times, either !

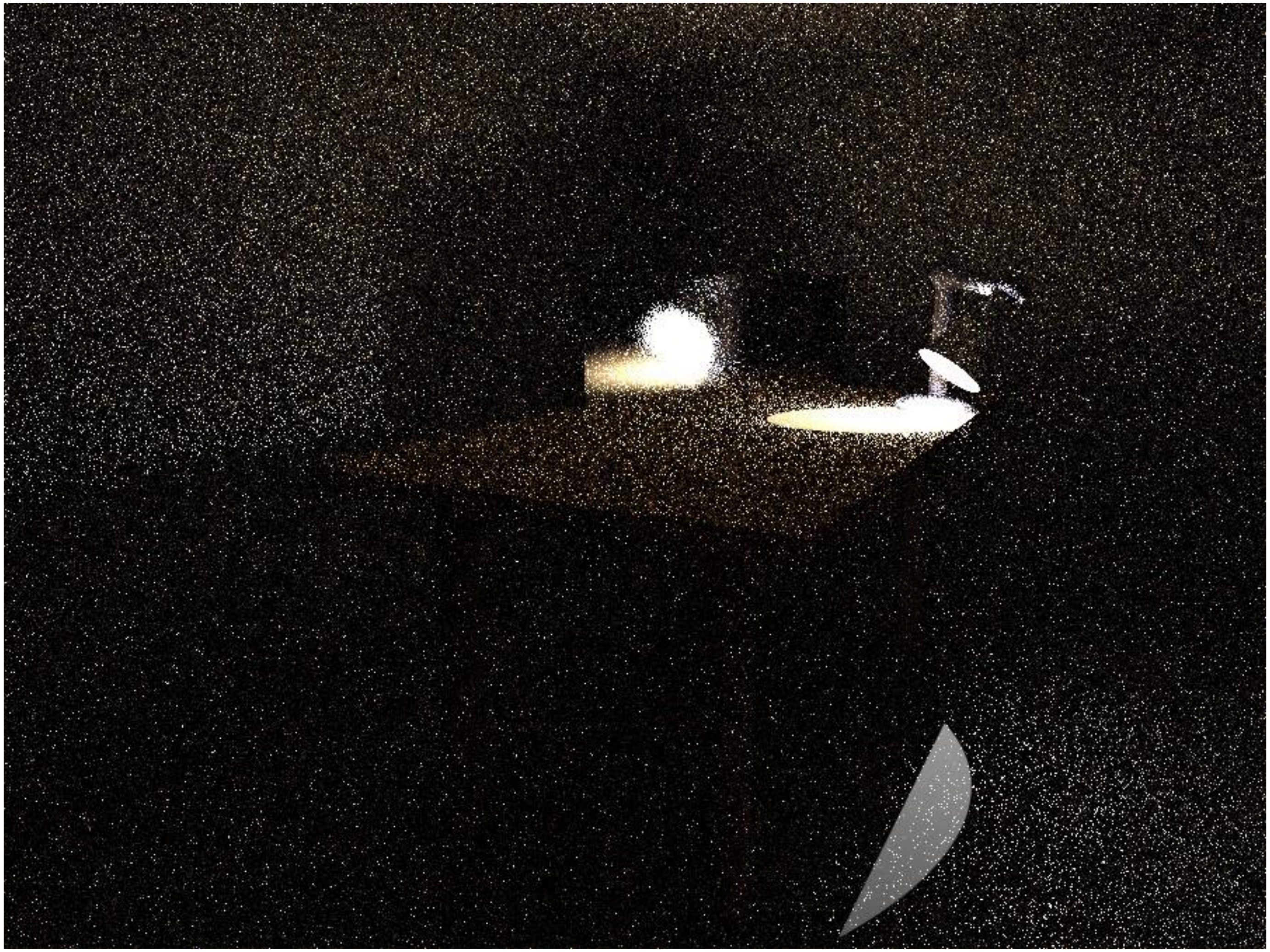
# Example

- **Light tracing (one eye ray, 1st generation only)**



# Example

- **Standard MC Path Tracing (same number of paths)**



# Example

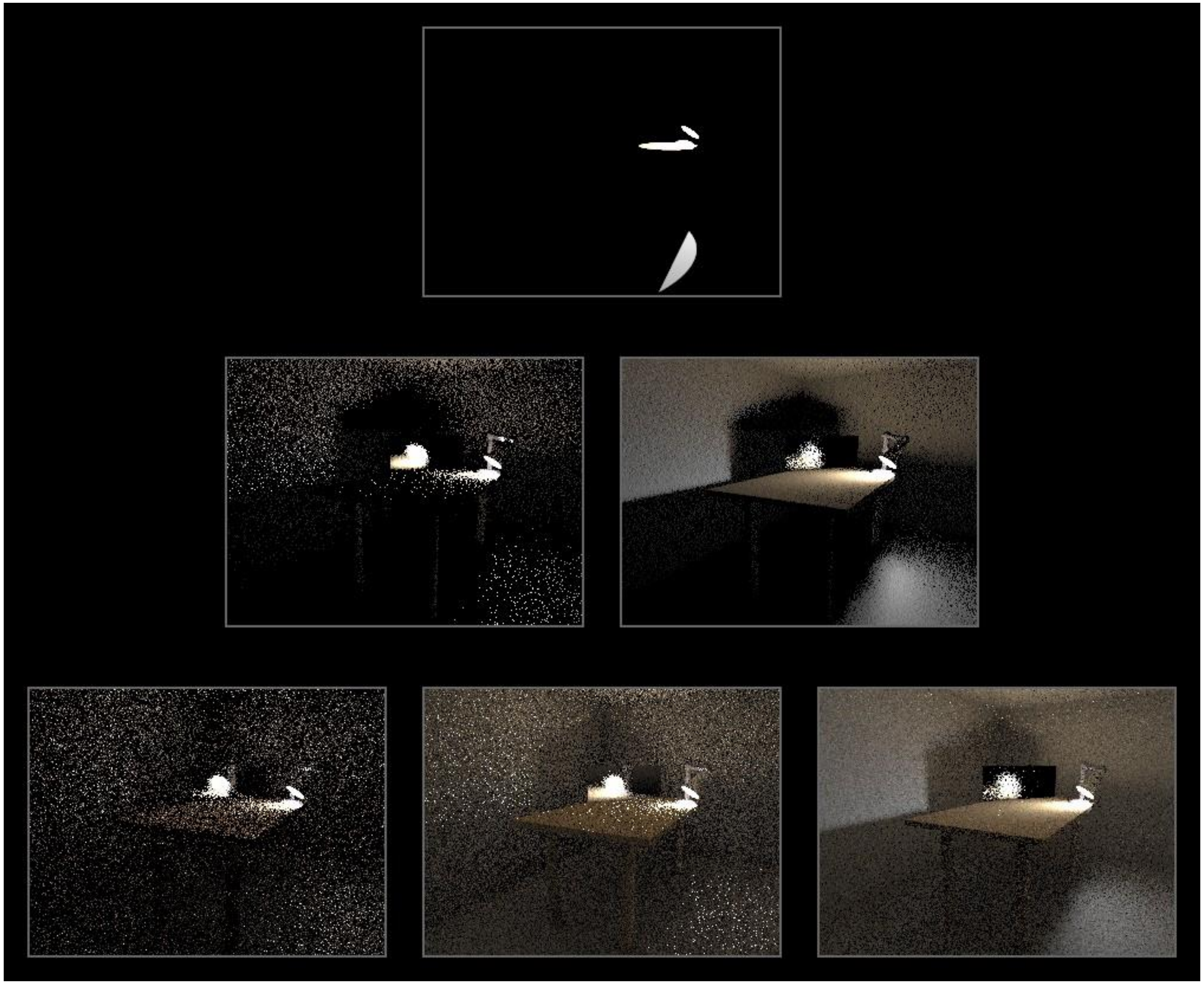
- **Contribution of Different Paths**

**[Not shown: direct connection eye to light + all from light]**

**One reflection**
One step from the eye (plus direct connection to light)

**Two reflections**
l: Two steps from the eye
r: One step from the eye, one step from light source

**Three reflections**
l: Three steps from the eye
m: two steps from the eye, one from light source
r: one step from the eye, two from the light sources

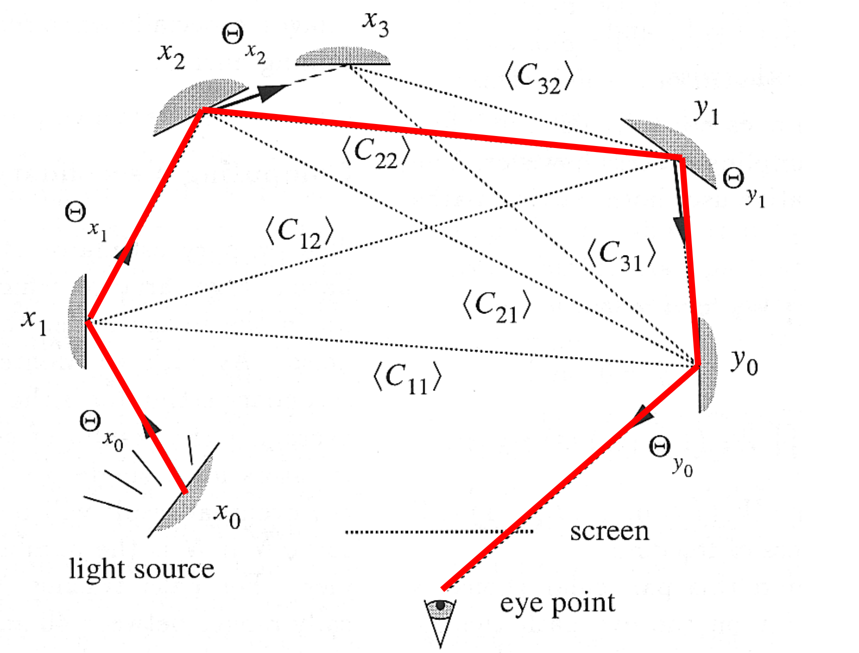


# Bidirectional Path-Tracing

- **Combination of Estimators**
  - Every option of generating a specific path $\pi$ defines its own estimator with given $p_{k,m}(\pi)$
  - Weighted MC sampling provides new combined estimator of a bi-directionally generated path

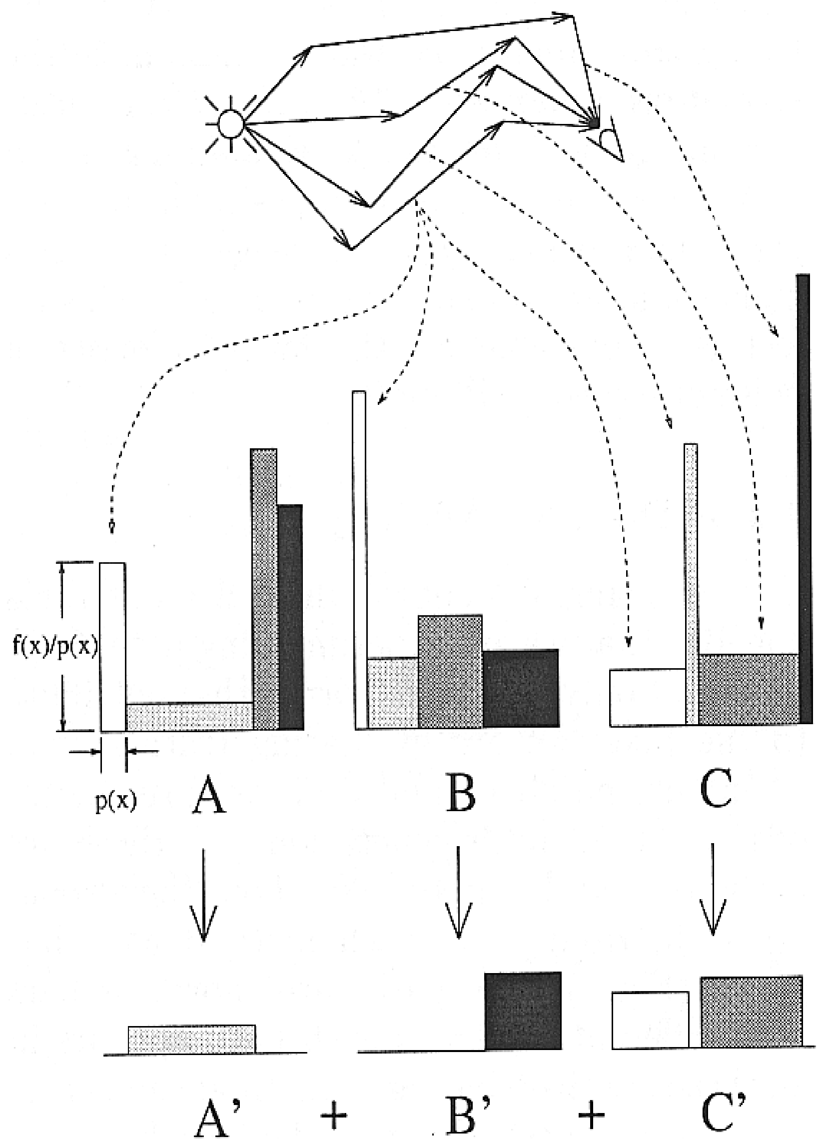
$$\overline{C} = \sum_{i=1}^{N_e} \sum_{j=1}^{N_l} w_{ij} \left\langle C_{ij} \right\rangle$$

  - $N_e$: # reflections on eye paths
  - $N_l$: # reflections on light paths
  - $w_{ij}$: weights for combination



# Combination of Estimators

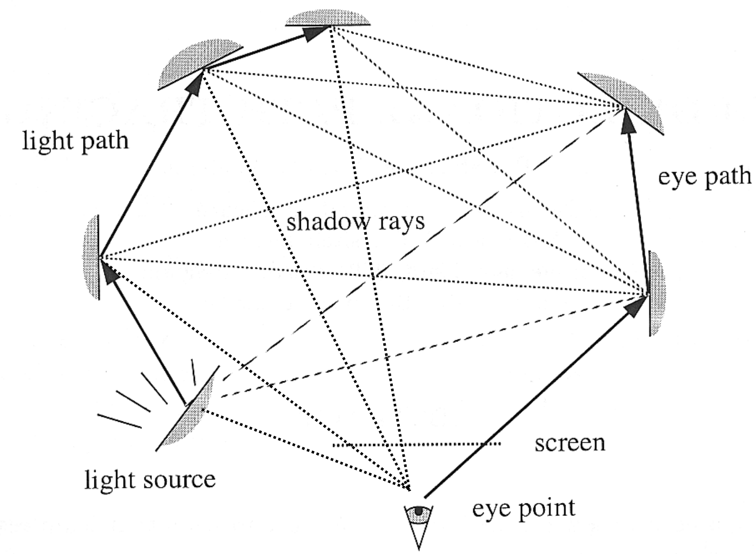
- **Example:**
  - Four paths between LS and eye
  - Weighted with three estimators
    - A, B, C
  - Selection with maximum heuristics
    - Choose $p_X(\pi)$ maximum
  - Area of rectangles is constant across A, B, C
    - f/p*p
  - Width corresponds to $p_X(\pi)$



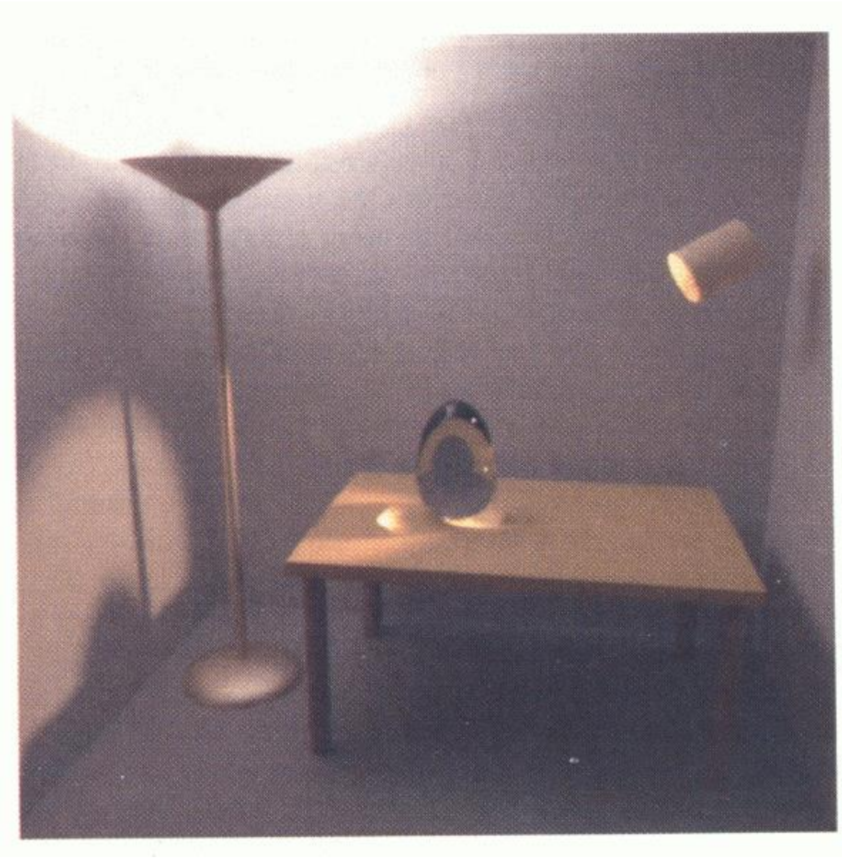
# Implementation

Example: Maximums Heuristics

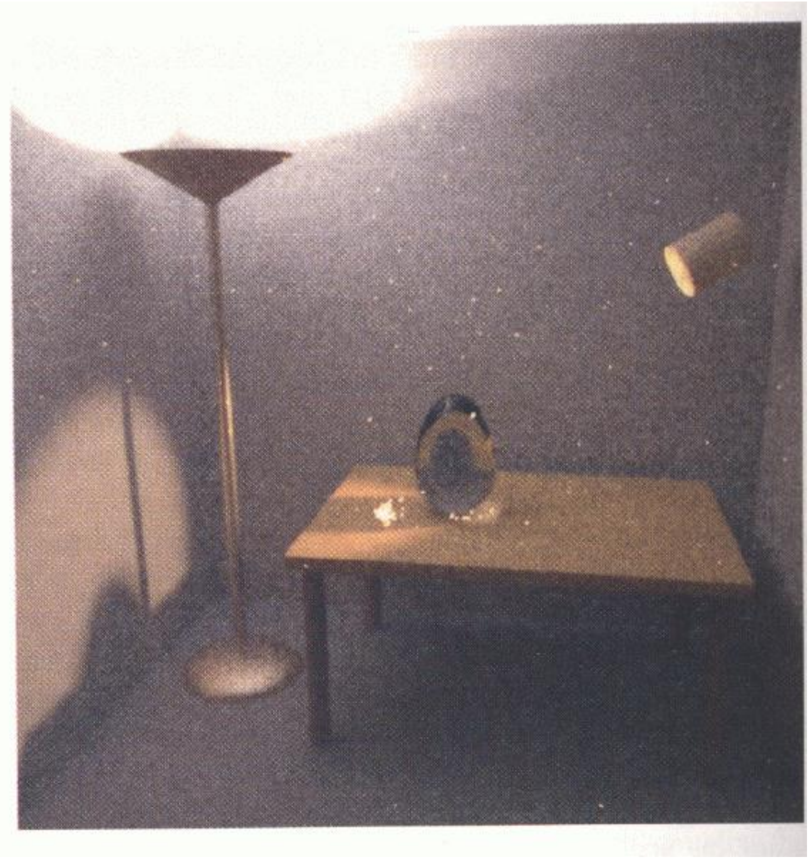
```
S= 0
P= GenerateBiDirPaths()
for light_segs= 0 to P.max_light_segments
  for eye_segs= 0 to P.max_eye_segments
    SP= ChooseSubPath(P, eye_segs, light_segs)
    // Compute best  estimator (Max-Heuristics)
    p= 0; segments= eye_segs + light_segs;
    // Iterate over different estimators:
    // assuming j segments generated
    // from camera
    for estimator= 0 to segments
      p_t= Probability(SP, estimator)
      if (p_t > p) p= p_t
    S= S + SP.f/p
return S
```



# Example



Bidirectional Path Tracing



Path Tracing

# Contributions of Different Paths

More camera segments

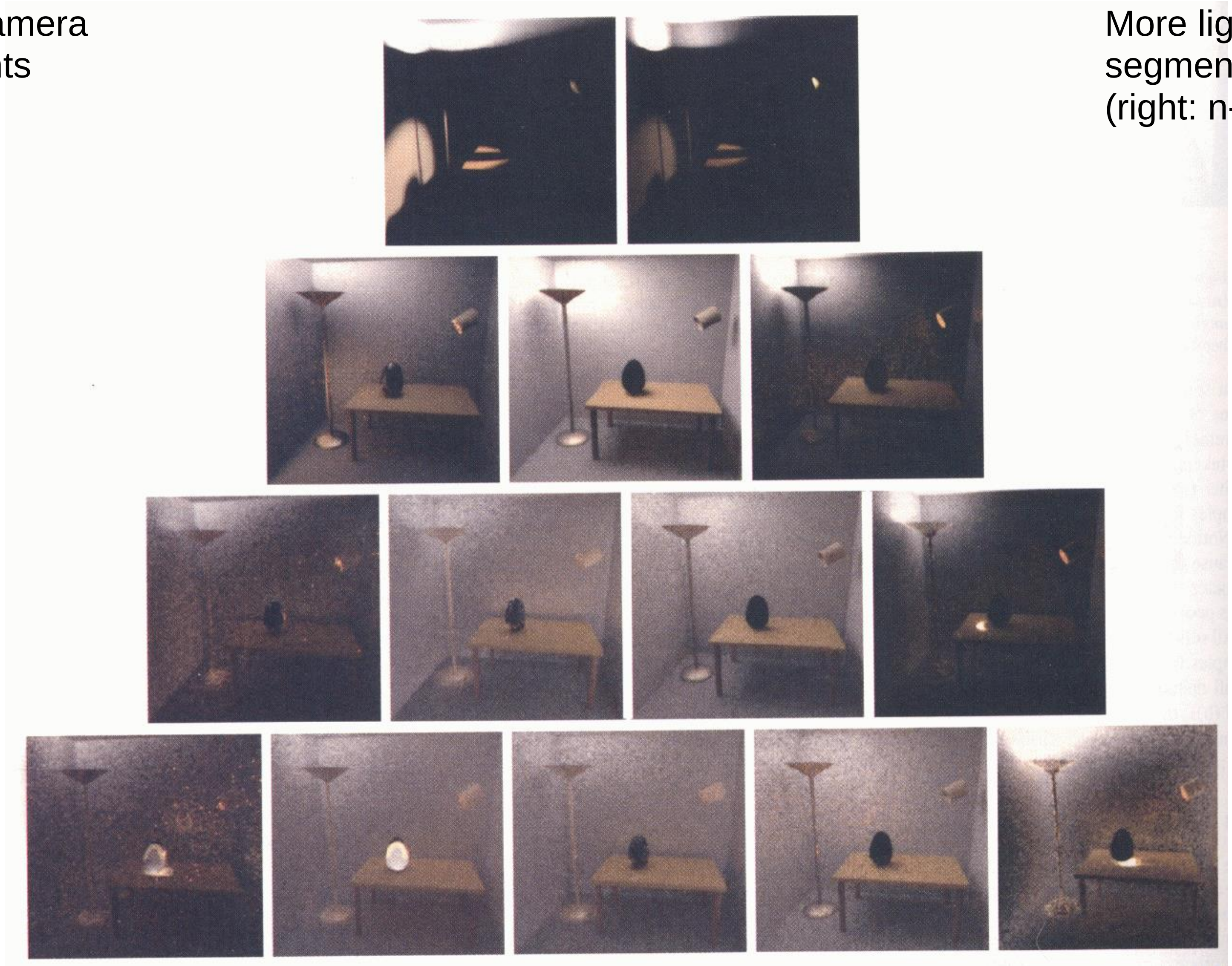
More light segments (right: n-1)

$p_{2,x}$

$p_{3,x}$

$p_{4,x}$

$p_{5,x}$



# Comparison w/ Path Tracing

- **Brute Force Method**
  - Only use $p_{n,0}$ method to generate paths
    - No points sampled from light source
  - Highly inefficient:
    - Probability of hitting the light is almost zero
    - Especially for point lights :-)
- **Path Tracing with Direct Lighting Optimization**
  - Next Event Estimation
  - Use $p_{n,0}$ and $p_{n,1}$ paths only
    - Path from the eye/camera plus direct connection to point sampled on light source

# NON-SYMMETRIC SCATTERING IN LIGHT TRANSPORT ALGORITHMS

# Use of Shading Normals

- **Shading Normals**
  - It is common to shade with respect to arbitrary normals
    - E.g. specified as normals at each triangle vertex
  - Allow many neat tricks
    - Smooth surface even though real surface is tessellated
    - Bump mapping, normal mapping, …
- **Problem**
  - Use of shading normals θ' is generally not energy conserving

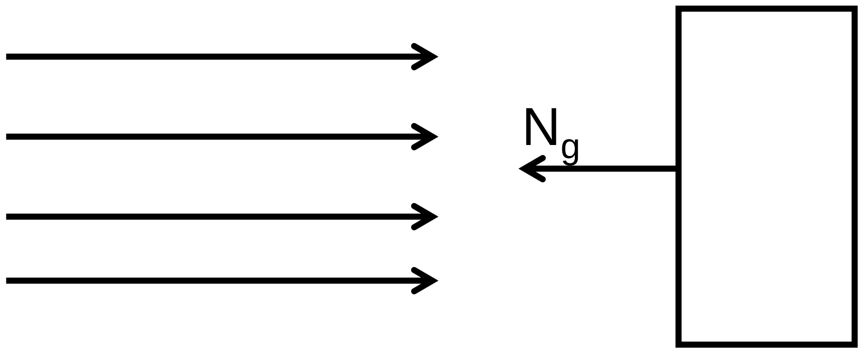
$$L_r = \int_{\Omega_+} f_r(\omega_o, x, \omega_i) \cos\theta_i \, d\omega_i$$

$$= \int_{\Omega_+} f_r'(\omega_o, x, \omega_i) \underbrace{\frac{\cos\theta_i'}{\cos\theta_i}}_{\text{can be arbitrarily large}} \cos\theta_i \, d\omega_i$$

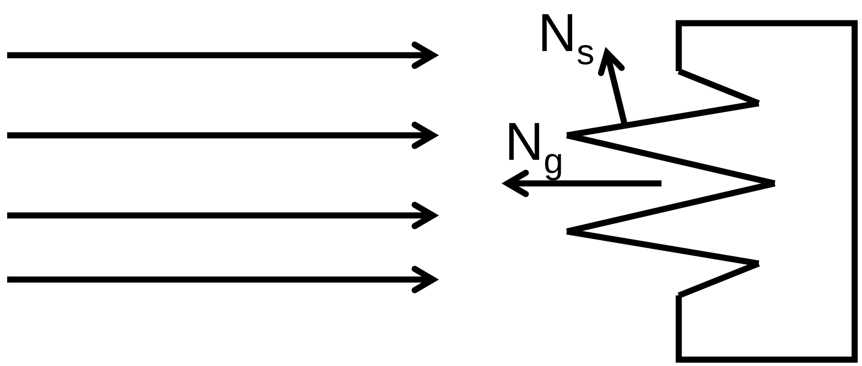
  - Can “generate” energy

# Use of Shading Normals

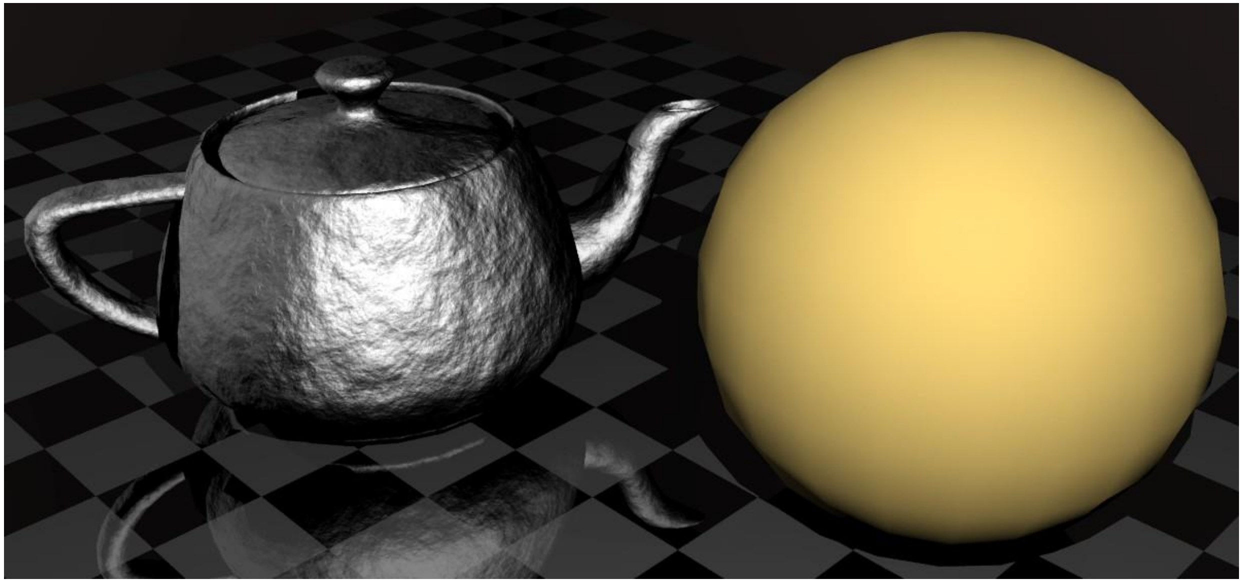
- **Energy "Generator"**
  - Light is received by an apparently small surface → some density



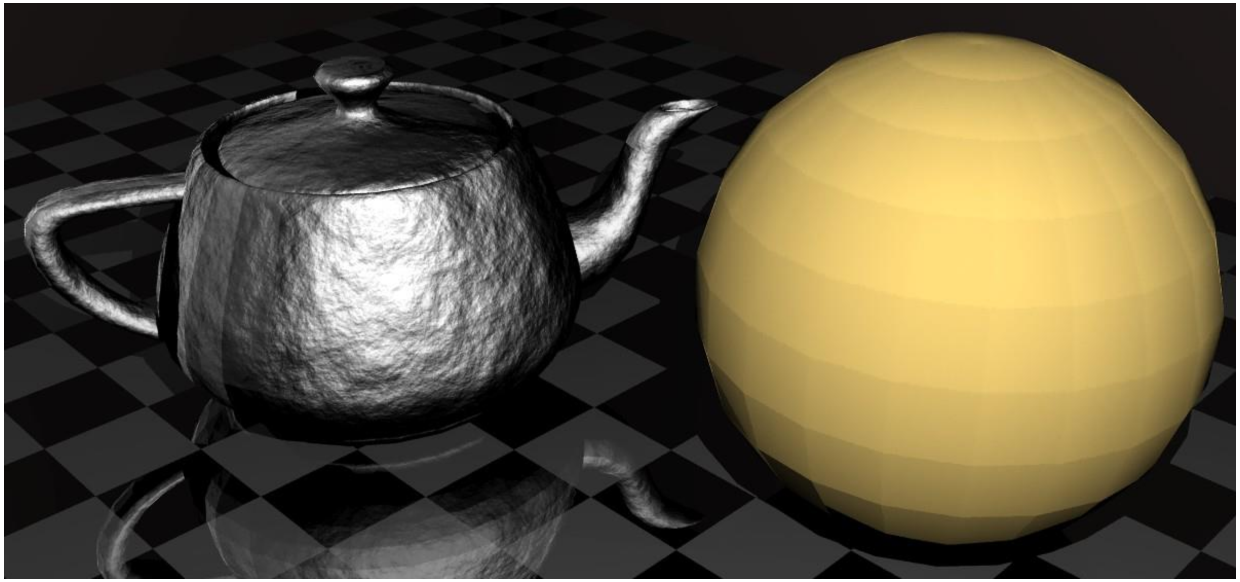

  - And emitted from an apparently much larger one, w/ same density



# Use of Shading Normals



Correct results



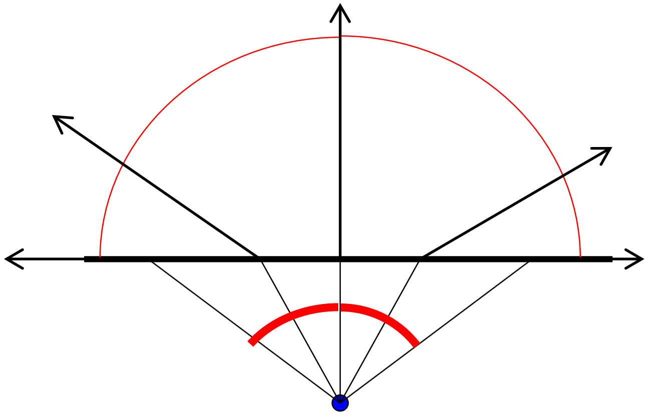
Wrong results

# Use of Shading Normals

- **Solution**
  - Unfortunately there seems to be no good solution to the problem
  - Except not using shading normals :-(
    - Or making them differ as little as possible from geometric normals

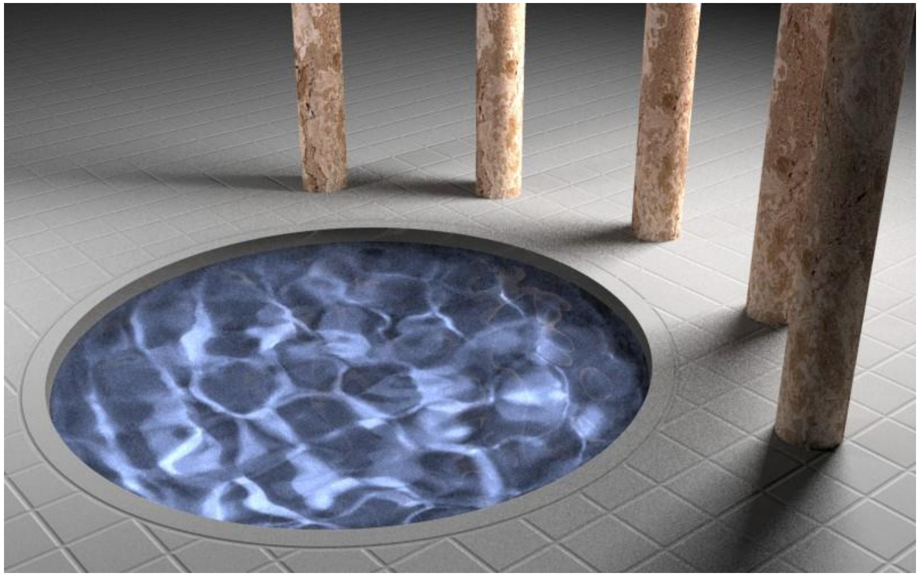
# Power versus Radiance

- **Light tracing and Refraction**
  - Distribution of “photons” carrying a certain energy/power
  - Power/energy does not change when photon is refracted
- **Ray Tracing and Refraction**
  - Consider
    - uniform illumination
    - a point below a refracting surface
  - If no light is absorbed at the surface then the same power comes through a smaller solid angle
    ➔ increased radiance

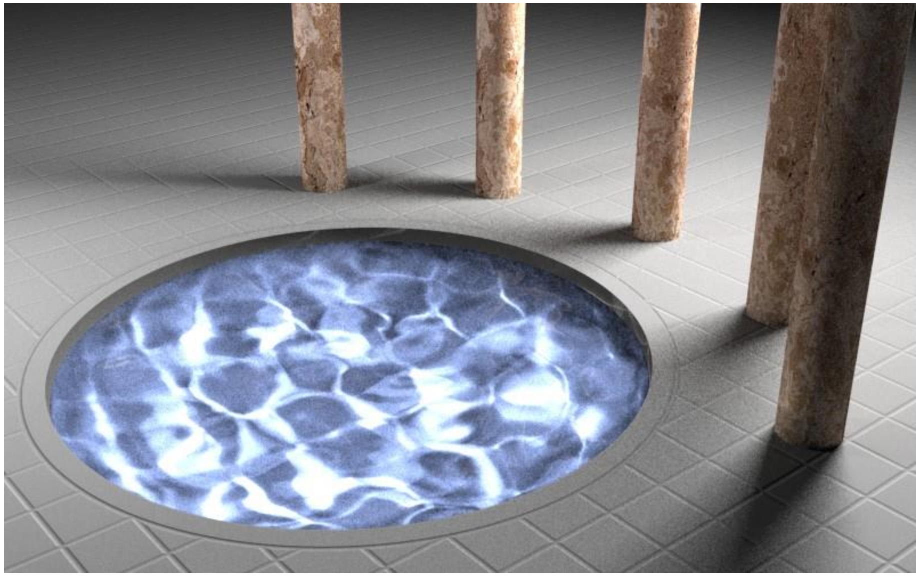



$$L_t = \frac{\eta_t^2}{\eta_i^2} L_i$$

# Power versus Radiance



Correct image rendered with particle tracing



Incorrect image rendered assuming the BRDF is symmetric also for refraction